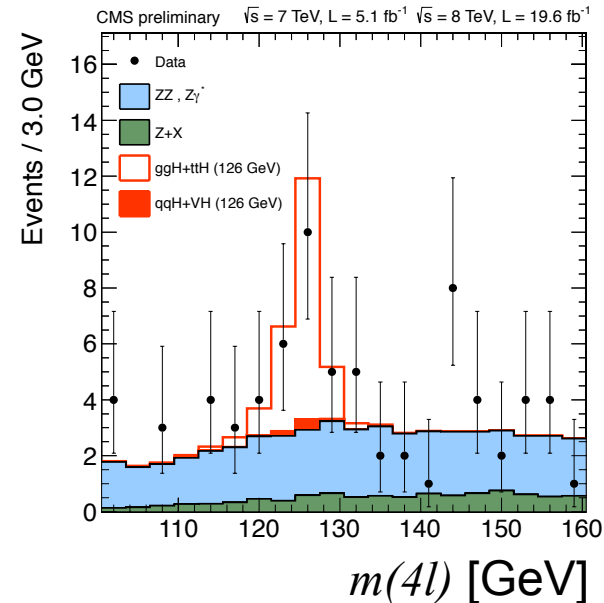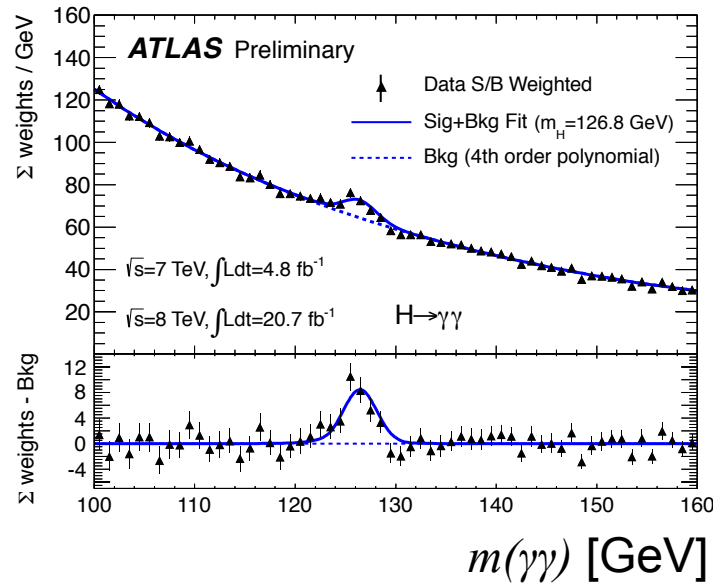G fitter

http://cern.ch/Gfitter

EPJC 72, 2205 (2012), arXiv:1209.2716

The ElectroWeak fit of Standard Model after the Discovery of the Higgs-like boson
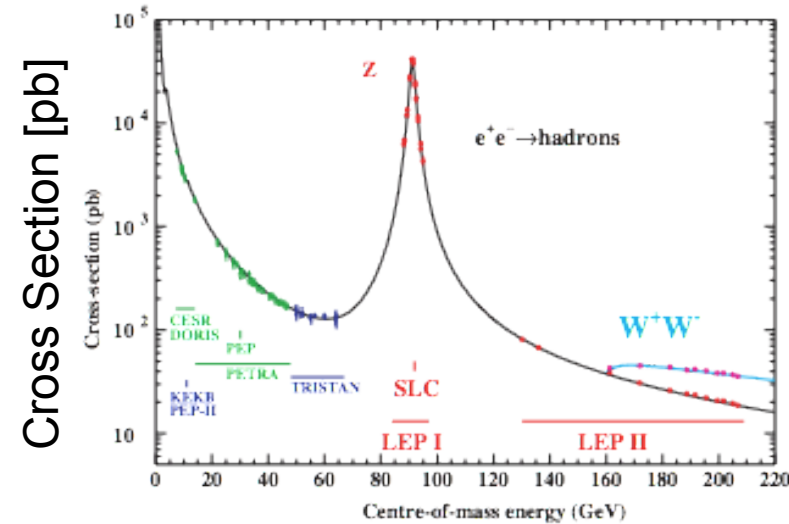
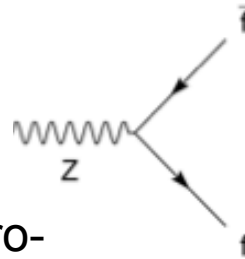(*) M. Baak, J. Haller, A. Höcker, R. Kogler, K. Mönig, M. Schott, J. Stelzer

- **Tree level relations for Z→f$\bar{\text{f}}$**
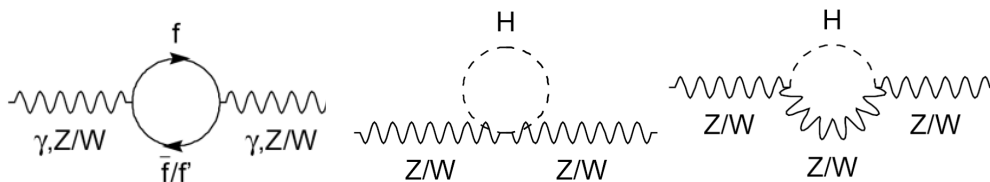
  -
  $$i\bar{f}\gamma^\mu \left(g_{V,f} - g_{A,f}\gamma_5\right) f Z_\mu$$

  - Unification connects the electro-magnetic and weak couplings
  - E.g. M_W can be expressed as function of M_Z and G_F

$\sqrt{s}$ [GeV]

- **The impact of radiative corrections**

  - Absorbed into EW form factors: ρ, κ, Δr
  - Effective couplings at the Z-pole
  - Quadraticly dependent on m_t, *logarithmic* dependence on M_H

$$g_{V,f} = \sqrt{\rho_Z^f} \left(I_3^f - 2Q^f \sin^2\theta_{\text{eff}}^f\right)$$

$$g_{A,f} = \sqrt{\rho_Z^f} I_3^f$$

$$\sin^2\theta_{\text{eff}}^f = \kappa_Z^f \sin^2\theta_W$$

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha(1+\Delta r)}{G_F M_Z^2}}\right)$$

- **EW fits: a long tradition**

  - Huge amount of pioneering work by many!

  - Precision measurements crucial, first from LEP/SLC, then Tevatron and now LHC.

  - Precise understanding of loop corrections essential.

    - Observables known at least at two-loop order, sometimes more.
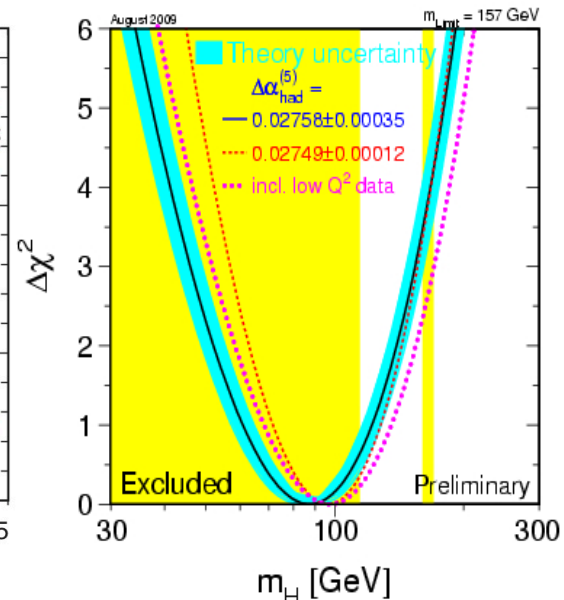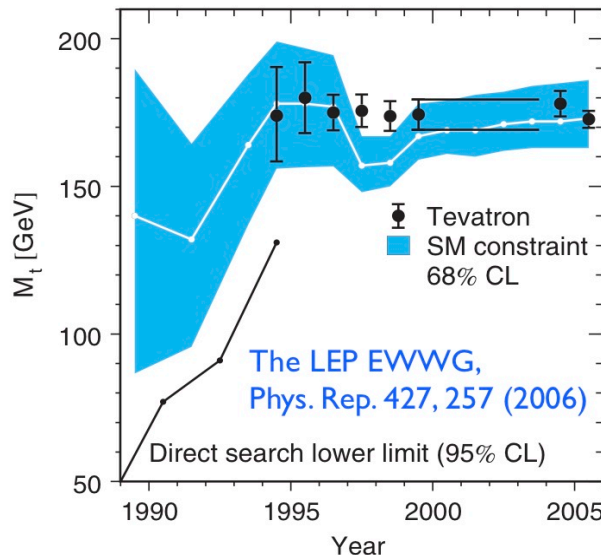
- **Hunt for the Higgs**

  - M_H last missing input parameter

  - Indirect determination from EW fit (2012): $M_H = 96^{+31}_{-24}$ GeV

  - (Direct exclusion limits also incorporated in EW fits.)
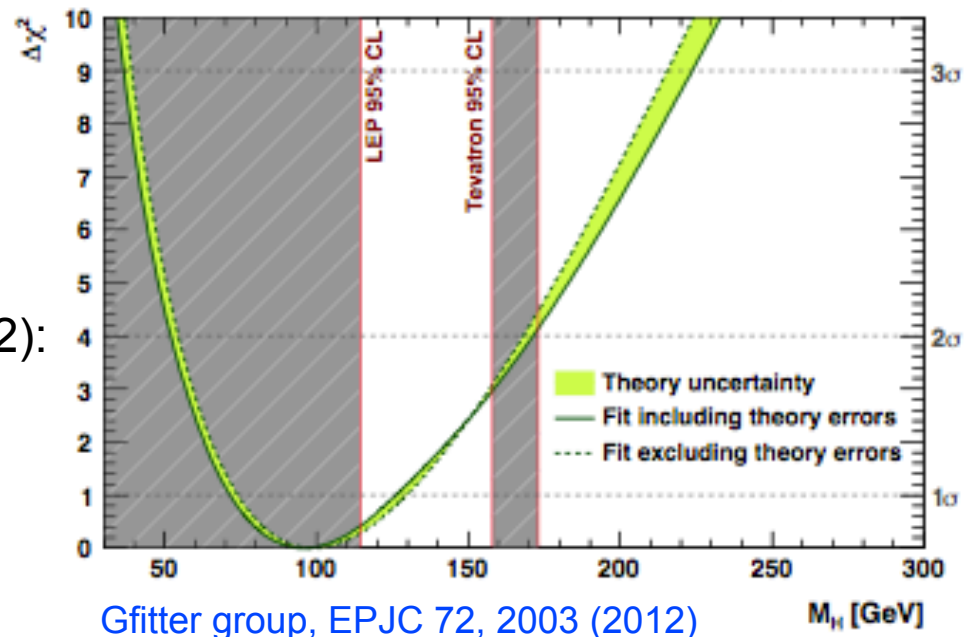
Gfitter group, EPJC 72, 2003 (2012)

The SM fit with Gfitter, including the Higgs

- Discovery of Higgs-like boson by LHC

  - Cross section and branching ratios sofar ~compatible with SM Higgs boson

  - This talk: assume boson is SM Higgs.

  - Use in EW fit: M_H = 125.7 ± 0.4 GeV

  - Change between fully uncorrelated and fully correlated systematic uncertainties is minor: δM_H : 0.4 → 0.5 GeV

- *For first time SM is fully over-constrained → test its self-consistency!*
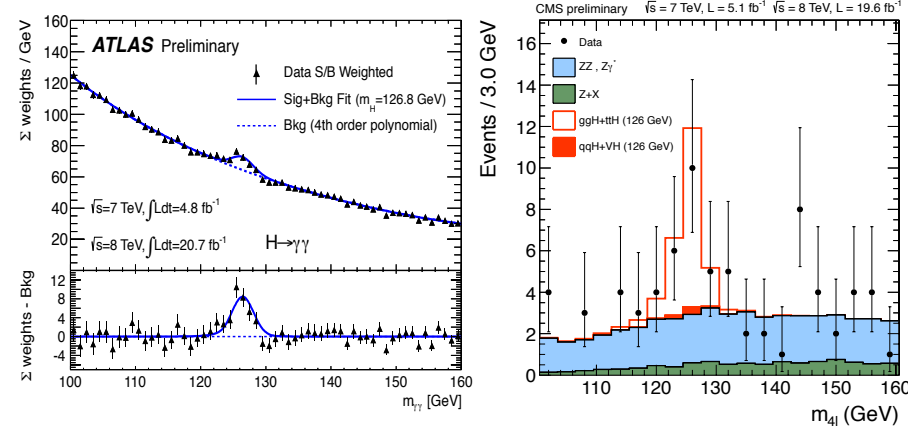
- In EW fit with Gfitter we use state-of-the-art calculations:

  - M_W Mass of the W boson   [M. Awramik et al., Phys. Rev. D69, 053006 (2004)]

  - Γ_Z , Γ_W Partial and total widths of the Z and W   [Cho et. al, arXiv:1104.1769]

  - $\sin^2\theta^\ell_{eff}$ Effective weak mixing angle   [M. Awramik et al., JHEP 11, 048 (2006), M. Awramik et al., Nucl.Phys.B813:174-187 (2009)]

  - Γ_{had} QCD Adler functions at N3LO   [P. A. Baikov et al., PRL108, 222003 (2012)]

  - R_b Partial width of $Z\to b\bar{b}$ [Freitas et al., JHEP08, 050 (2012)]   ⟵ **New! full 2-loop calc.**

Electroweak fit – Experimental inputs

- **Latest experimental inputs:**

  - **Z-pole observables**: from LEP / SLC
    [ADLO+SLD, Phys. Rept. 427, 257 (2006)]

  - **M_W and Γ_W** from LEP/Tevatron
    [arXiv:1204.1069]

  - **m_{top}** : average from Tevatron
    [arXiv:1207.1069]

  - **m_c, m_b** world averages
    [PDG, J. Phys. G33,1 (2006)]

  - **$\Delta\alpha_{had}^{(5)}(M_Z^2)$** including α_S dependency
    [Davier et al., EPJC 71, 1515 (2011)]

  - **M_H** from LHC
    [arXiv:1207.7214, arXiv:1207.7235]

- **7 free fit parameters:**

  - M_Z, M_H, $\alpha_S(M_Z^2)$, $\Delta\alpha_{had}^{(5)}(M_Z^2)$, m_t, $\overline{m}_c$, $\overline{m}_b$

  - Two nuisance parameters for theoretical uncertainties:
    δM_W (4 MeV), $\delta sin^2\theta^l_{eff}$ (4.7x10^{-5})

|		
M_H [GeV]$^{(\circ)}$	125.7 ± 0.4	**LHC**
M_W [GeV]	80.385 ± 0.015	**Tevatron**
Γ_W [GeV]	2.085 ± 0.042	
M_Z [GeV]	91.1875 ± 0.0021	
Γ_Z [GeV]	2.4952 ± 0.0023	
σ_{had}^0 [nb]	41.540 ± 0.037	**LHC**
R_ℓ^0	20.767 ± 0.025	
$A_{FB}^{0,\ell}$	0.0171 ± 0.0010	
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	**SLC**
$\sin^2\theta_{eff}^\ell(Q_{FB})$	0.2324 ± 0.0012	
A_c	0.670 ± 0.027	**SLC**
A_b	0.923 ± 0.020	
$A_{FB}^{0,c}$	0.0707 ± 0.0035	**LEP**
$A_{FB}^{0,b}$	0.0992 ± 0.0016	
R_c^0	0.1721 ± 0.0030	
R_b^0	0.21629 ± 0.00066	
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]	173.18 ± 0.94	**Tevatron**
$\Delta\alpha_{had}^{(5)}(M_Z^2)$ $^{(\triangle\triangledown)}$	2757 ± 10	

- From the Gfitter Group, EPJC 72, 2205 (2012)

- Left: full fit incl. M_H

- Middle: fit not incl. M_H

- Right: fit incl M_H, not the row

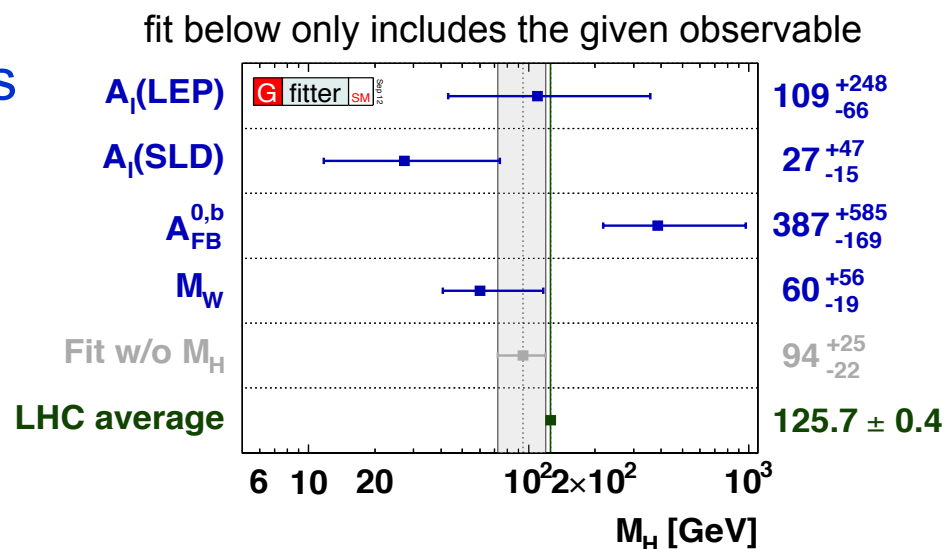| Parameter	Input value	Free in fit	Fit result incl. M_H	Fit result not incl. M_H	Fit result incl. M_H but not exp. input in row
M_H [GeV]$^{(\circ)}$	125.7 ± 0.4	yes	125.7 ± 0.4	94^{+25}_{-22}	94^{+25}_{-22}
M_W [GeV]	80.385 ± 0.015	–	80.367 ± 0.007	80.380 ± 0.012	80.359 ± 0.011
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.092 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1878 ± 0.0021	91.1874 ± 0.0021	91.1983 ± 0.0116
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4954 ± 0.0014	2.4958 ± 0.0015	2.4951 ± 0.0017
σ^0_{had} [nb]	41.540 ± 0.037	–	41.479 ± 0.014	41.478 ± 0.014	41.470 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.740 ± 0.017	20.743 ± 0.018	20.716 ± 0.026
$A^{0,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	–	0.01627 ± 0.0002	0.01637 ± 0.0002	0.01624 ± 0.0002
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	$0.1473^{+0.0006}_{-0.0008}$	0.1477 ± 0.0009	$0.1468 \pm 0.0005^{(\dagger)}$
$\sin^2\theta^\ell_{\mathrm{eff}}(Q_{\mathrm{FB}})$	0.2324 ± 0.0012	–	$0.23148^{+0.00011}_{-0.00007}$	$0.23143^{+0.00010}_{-0.00012}$	0.23150 ± 0.00009
A_c	0.670 ± 0.027	–	$0.6680^{+0.00025}_{-0.00038}$	$0.6682^{+0.00042}_{-0.00035}$	0.6680 ± 0.00031
A_b	0.923 ± 0.020	–	$0.93464^{+0.00004}_{-0.00007}$	0.93468 ± 0.00008	0.93463 ± 0.00006
$A^{0,c}_{\mathrm{FB}}$	0.0707 ± 0.0035	–	$0.0739^{+0.0003}_{-0.0005}$	0.0740 ± 0.0005	0.0738 ± 0.0004
$A^{0,b}_{\mathrm{FB}}$	0.0992 ± 0.0016	–	$0.1032^{+0.0004}_{-0.0006}$	0.1036 ± 0.0007	0.1034 ± 0.0004
R^0_c	0.1721 ± 0.0030	–	0.17223 ± 0.00006	0.17223 ± 0.00006	0.17223 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21474 ± 0.00003	0.21475 ± 0.00003	0.21473 ± 0.00003
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	$1.27^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	$4.20^{+0.17}_{-0.07}$	–
m_t [GeV]	173.18 ± 0.94	yes	173.52 ± 0.88	173.14 ± 0.93	$175.8^{+2.7}_{-2.4}$
$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ $^{(\triangle\triangledown)}$	2757 ± 10	yes	2755 ± 11	2757 ± 11	2716^{+49}_{-43}
$\alpha_s(M_Z^2)$	–	yes	0.1191 ± 0.0028	0.1192 ± 0.0028	0.1191 ± 0.0028
$\delta_{\mathrm{th}}M_W$ [MeV]	$[-4,4]_{\mathrm{theo}}$	yes	4	4	–
$\delta_{\mathrm{th}}\sin^2\theta^\ell_{\mathrm{eff}}$ $^{(\triangle)}$	$[-4.7,4.7]_{\mathrm{theo}}$	yes	-1.4	4.7	–
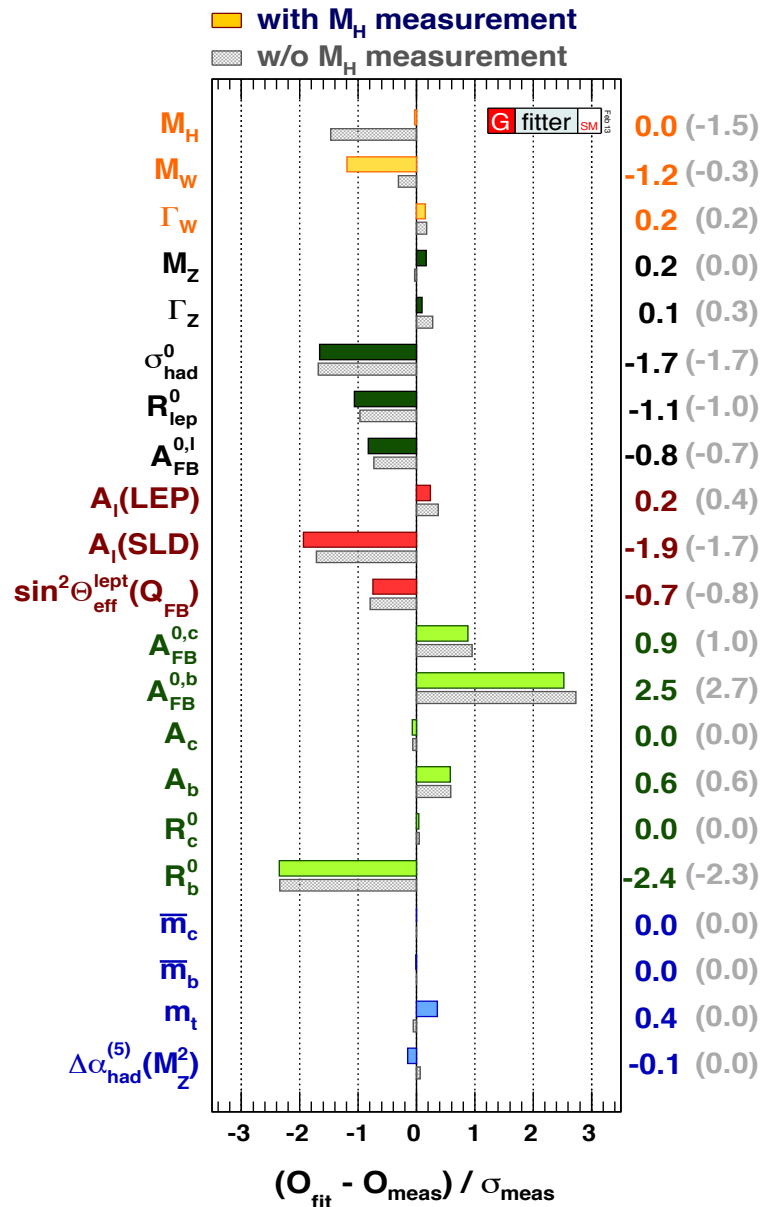
Plot inspired by Eberhardt et al. [arXiv:1209.1101]

- Pull values of full fit (with M_H)

  - No individual value exceeds 3σ

  - Small pulls for M_H, M_Z, $\Delta\alpha_{had}^{(5)}(M_Z^2)$, $\overline{m}_c$, $\overline{m}_b$ indicate that input accuracies exceed fit requirements

  - Largest deviations in b-sector: $A^{0,b}_{FB}$ and R^0_b with 2.5σ and -2.4σ → largest contribution to χ^2

  - R^0_b using one-loop calculation -0.8σ

    - R^0_b has only little dependence on M_H

- Goodness of fit – p-value:

  - $\chi^2_{min} = 21.8$ → Prob(χ^2_{min}, 14) = 8 %

  - From pseudo experiments: 7±1%

    - Large value of χ^2_{min} not due to inclusion of M_H measurement.

    - Without M_H measurement: $\chi^2_{min} = 20.3$ → Prob(χ^2_{min}, 13) = 9%

Higgs results of the EW fit

- **Scan of $\Delta\chi^2$ profile versus M_H**
  - Grey band: fit w/o M_H measurement
  - Blue line: full SM fit, with M_H meas.
  - Fit w/o M_H measurement gives:
    $M_H = 94^{+25}_{-22}$ GeV
  - Consistent at 1.3σ with LHC measurement.
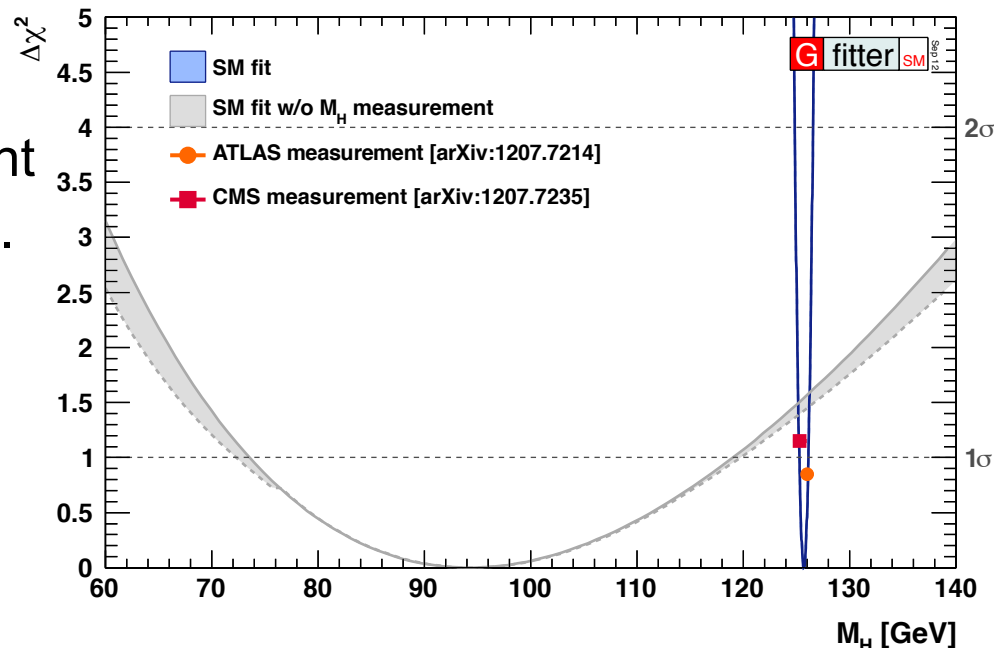

- **Bottom plot: impact of other most sensitive Higgs observables**
  - Determination of M_H removing all sensitive observables except the given one.
  - Known tension (2.5σ) between $A_l(SLD)$, $A^{0,b}_{FB}$, and M_W clearly visible.

fit below only includes the given observable

Indirect determination of W mass

- Scan of $\Delta\chi^2$ profile versus M_W

  - Also shown: SM fit with minimal inputs: M_Z, G_F, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, M_H, and fermion masses

  - Good consistency between total fit and SM w/ minimal inputs

- M_H measurement allows for precise constraint on M_W

  - Agreement at 1.4σ

- Fit result for indirect determination of M_W (full fit w/o M_W):
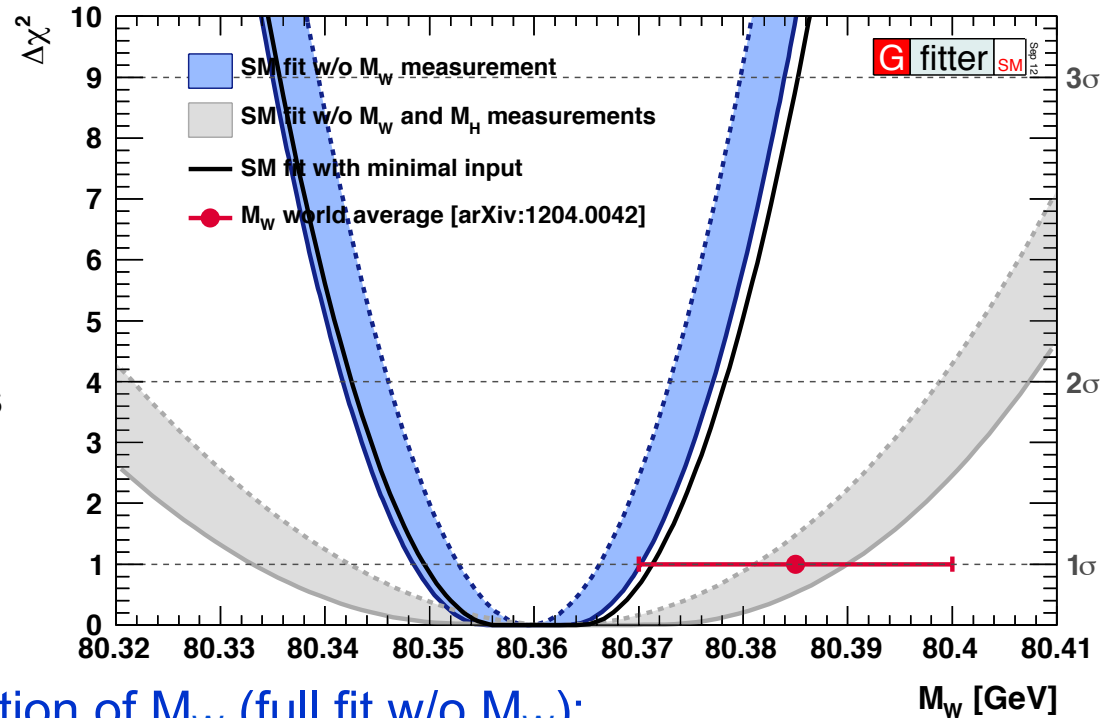
$$M_W = 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{had}}$$
$$\pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm 0.0040_{theo}$$
$$= 80.359 \pm 0.011_{tot} ,$$

- **More precise estimate of M_W than the direct measurements!**

  - Uncertainty on world average measurement: 15 MeV

Indirect effective weak mixing angle

- Right: scan of $\Delta\chi^2$ profile versus $\sin^2\theta_{eff}^l$
  - All sensitive measurements removed from the SM fit.
  - Also shown: SM fit with minimal inputs

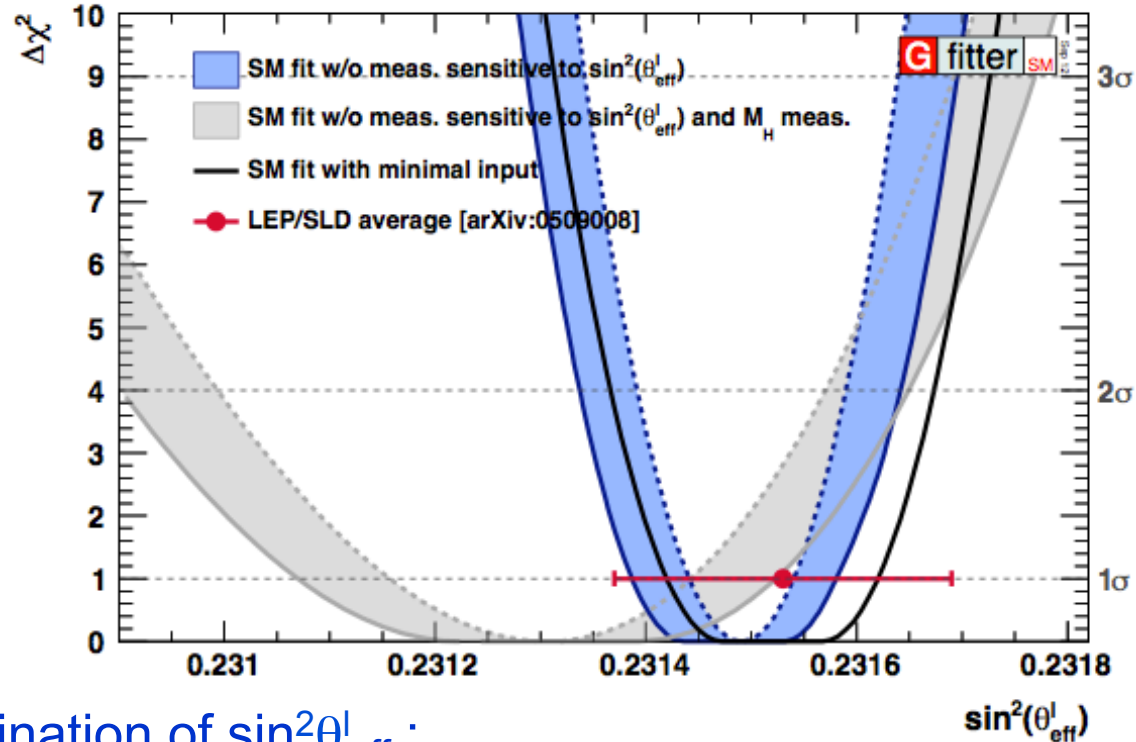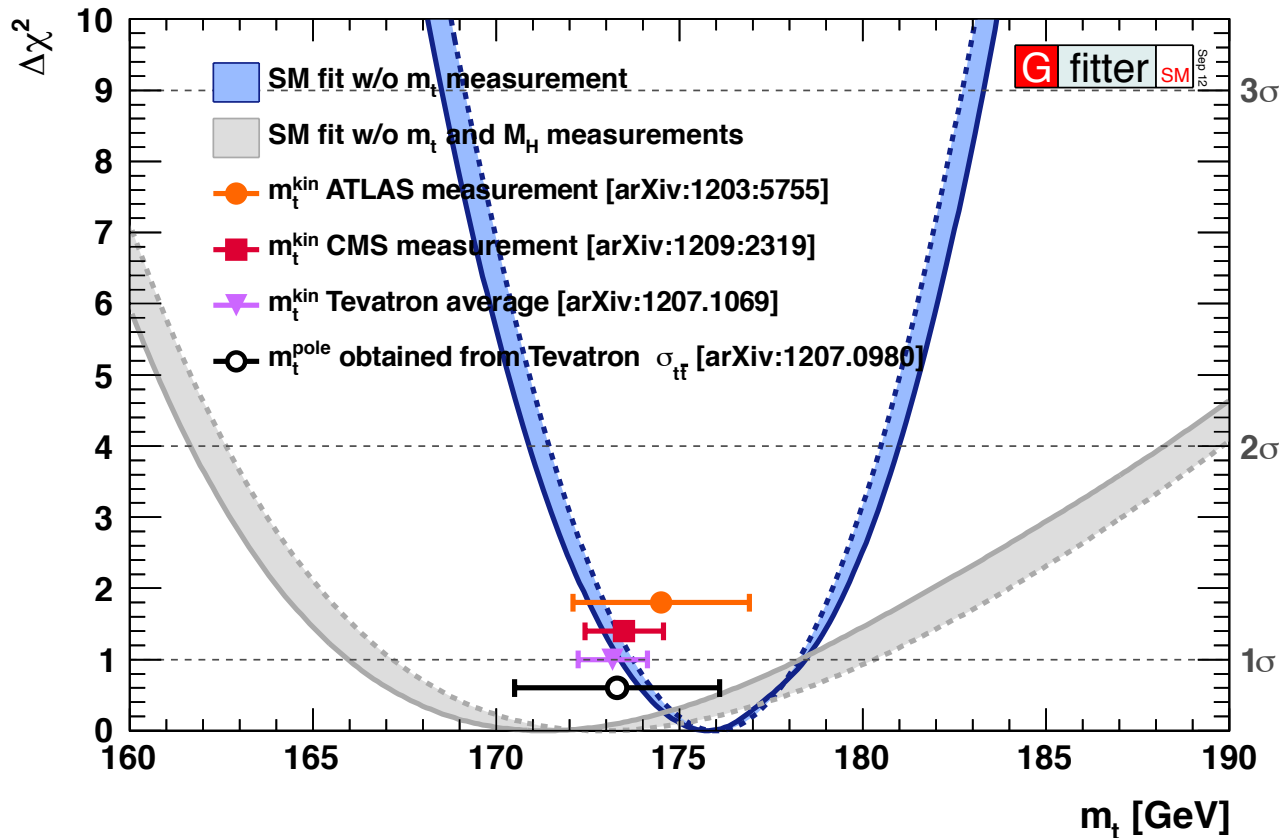- M_H measurement allows for very precise constraint on $\sin^2\theta_{eff}^l$

- Fit result for indirect determination of $\sin^2\theta_{eff}^l$:

$$\sin^2\theta_{eff}^\ell = 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{had}}$$
$$\pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{theo} ,$$
$$= 0.23150 \pm 0.00010_{tot} ,$$

- More precise than direct determination (from LEP/SLD) !
  - Uncertainty on LEP/SLD average: 1.6×10^{-4}

Indirect determination of top mass

- **Shown: scan of $\Delta\chi^2$ profile versus m_t (without m_t measurement)**
  - M_H measurement allows for significant better constraint of m_t
  - Indirect determination consistent with direct measurements
  - Indirect result: $m_t = 175.8^{+2.7}_{-2.4}$ GeV   (Tevatron average: 173.2 ± 0.9 GeV)

State of the SM: W versus top mass

- Scan of M_W vs m_t, with the direct measurements excluded from the fit.
- Results from Higgs measurement significantly reduces allowed indirect parameter space → corners the SM!
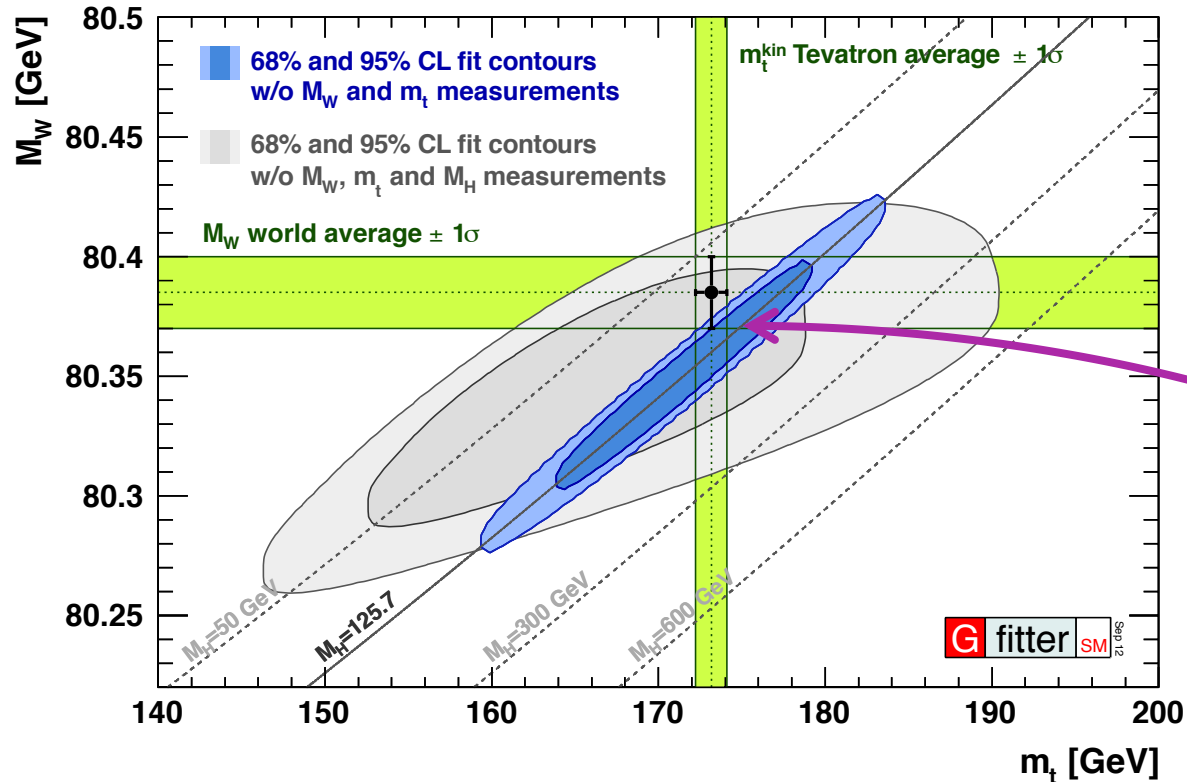
- Observed agreement demonstrates impressive consistency of the SM!

Constraints on S, T, U

- Electroweak fit sensitive to BSM physics through vacuum polarization corrections (also absorbed in ρ, κ, Δr).

- Described with STU parametrization
  [Peskin and Takeuchi, Phys. Rev. D46, 1 (1991)]

- SM: M_H = 125.7 GeV, m_t = 173.2 GeV
  - This defines (S,T,U) = (0,0,0)

- S, T depend logarithmically on M_H

- Fit result:

  S = 0.03 ± 0.10
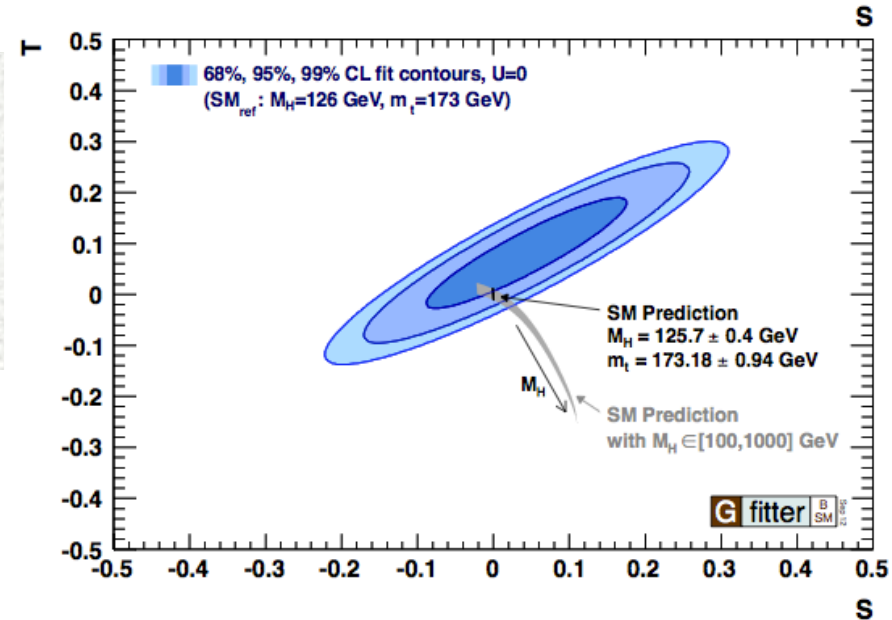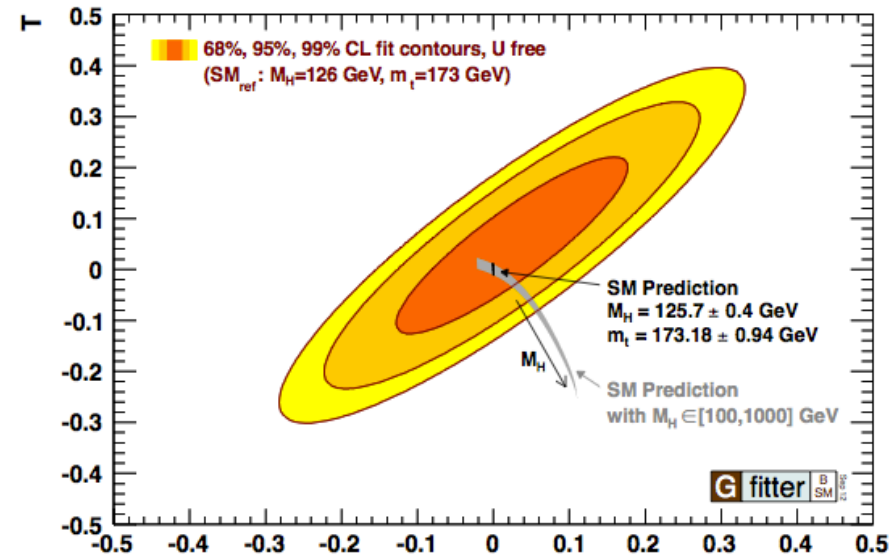
  T = 0.05 ± 0.12

  U = 0.03 ± 0.10

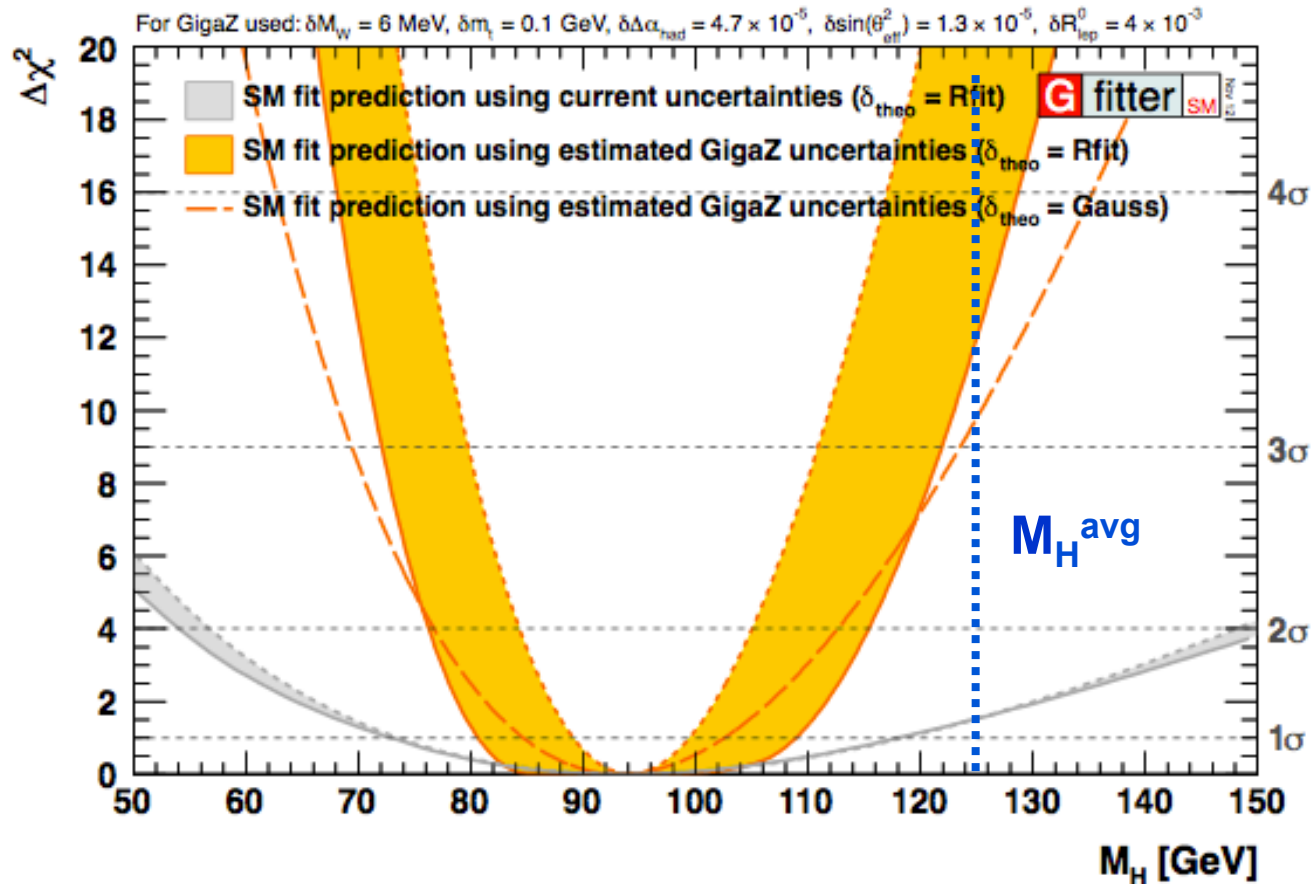 | | S | T | U |
  |---|-----|-----|-----|
  | S | 1 | +0.89 | -0.54 |
  | T | | 1 | -0.83 |
  | U | | | 1 |

- Stronger constraints from fit with U=0

- No indication for new physics.

- Can now use this constrain 4th gen, Ex-Dim, T-C, Higgs couplings, etc.

Prospects for ILC with Giga Z

- **Future Linear Collider could improve precision of EW observables tremendously.**

  - *WW threshold, to obtain M_W*
    - from threshold scan: $\delta M_W : 15 \rightarrow 6$ MeV
  - *ttbar threshold, to obtain m_t*
    - obtain m_t indirectly from production cross section: $\delta m_t : 0.9 \rightarrow 0.1$ GeV
  - *Z pole measurements*
    - High statistics: 10^9 Z decays: $\delta R^0_{lep} : 2.5 \cdot 10^{-2} \rightarrow 4 \cdot 10^{-3}$
    - With polarized beams, uncertainty on $\delta A^{0,f}_{LR}$: $10^{-3} \rightarrow 10^{-4}$, which translates to $\delta \sin^2\theta^l_{eff} : 1.6 \cdot 10^{-4} \rightarrow 1.3 \cdot 10^{-5}$

- **Low-energy data results**

  - *For $\Delta\alpha_{had}$:*
    - more precise e^+e^- cross section results for low energy ($\sqrt{s}$ < 1.8 GeV) and around cc resonance (KLOE-II, BaBar-ISR, BES-III), improved α_s, improvements in theory: $\Delta\alpha_{had}$: $10^{-4} \rightarrow 5 \cdot 10^{-5}$

For GigaZ used: $\delta M_W = 6$ MeV, $\delta m_t = 0.1$ GeV, $\delta\Delta\alpha_{had} = 4.7 \times 10^{-5}$, $\delta\sin(\theta_{eff}^2) = 1.3 \times 10^{-5}$, $\delta R_{lep}^0 = 4 \times 10^{-3}$

- SM fit prediction using current uncertainties (δ_{theo} = Rfit)
- SM fit prediction using estimated GigaZ uncertainties (δ_{theo} = Rfit)
- SM fit prediction using estimated GigaZ uncertainties (δ_{theo} = Gauss)

M_H^{avg}

- Logarithmic dependency on M_H → cannot compete with direct M_H meas.
- Indirect prediction M_H dominated by theory uncertainties.
  - No theory uncertainty: $M_H = 94.2^{+5.3}_{-5.0}$ GeV
  - Rfit scheme: $M_H = 92.3^{+16.6}_{-11.6}$ GeV

Prospects for ILC with Giga Z
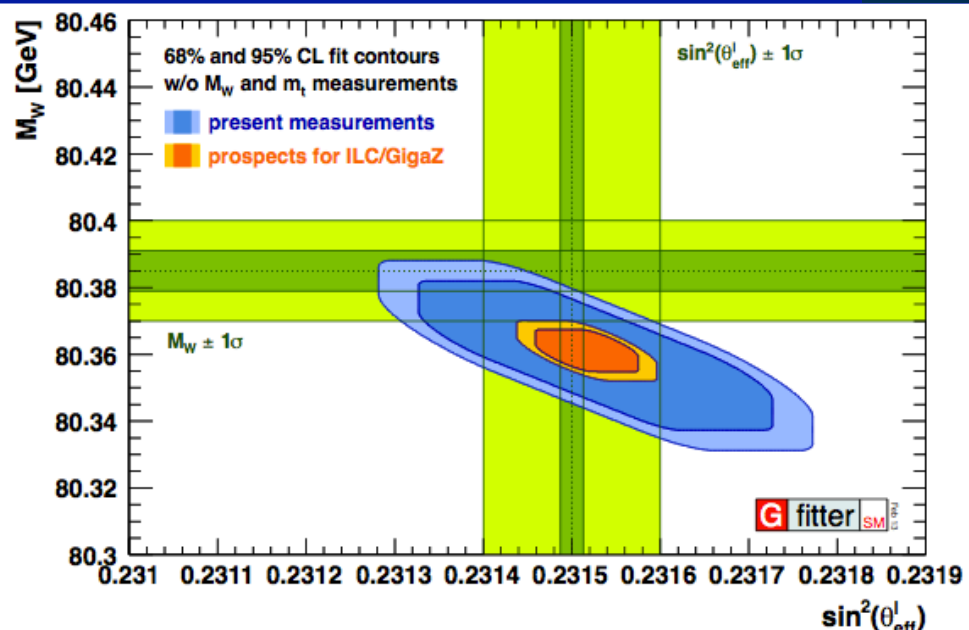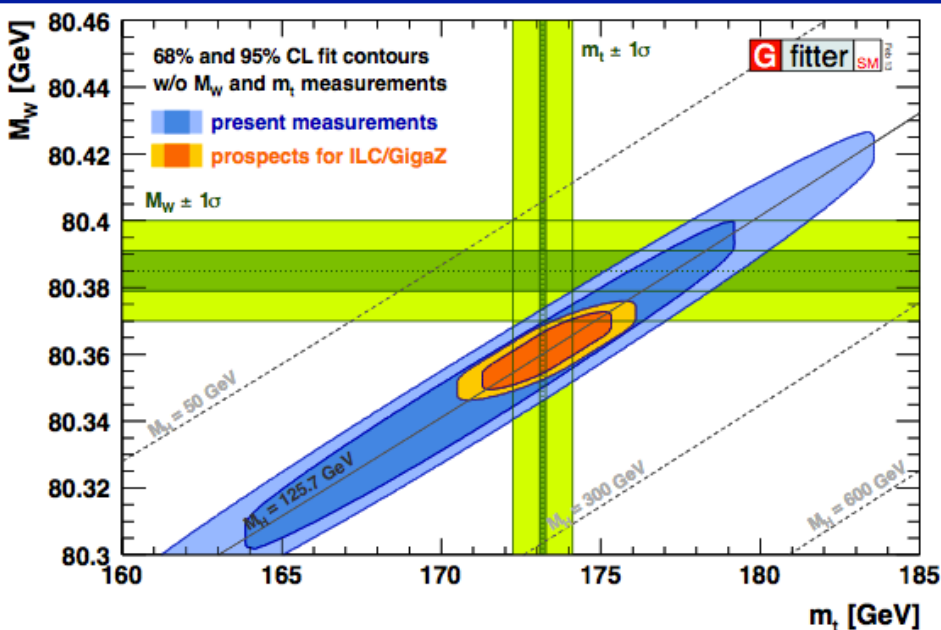
current precision
prospects for direct ILC measurements

- *Assuming also 50% of today's theoretical uncertainties*
  - Implies three-loop EW calculations!
- Huge reduction of uncertainty on indirect determinations
- Also strong constraints on S, T, U

Conclusion and Today's Prospects
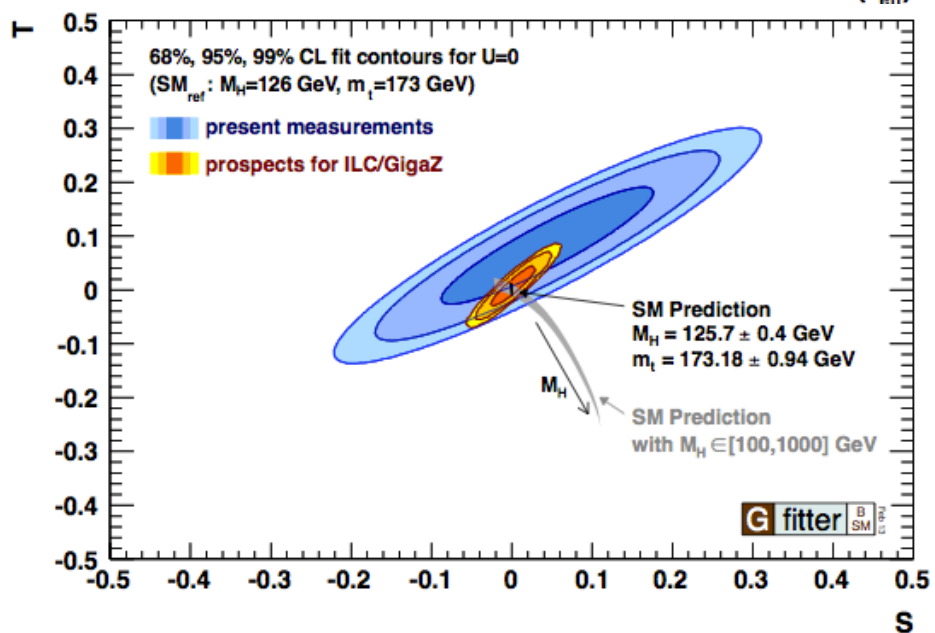
- Including M_H measurement, for first time SM is fully over-constrained!
  - M_H consistent at 1.3σ with indirect prediction from EW fit.

- p-Value of global electroweak fit of SM: 7% (pseudo-experiments)
  - Would be great to revisit $Z \to b\bar{b}$, both theoretically and experimentally

- Knowledge of M_H dramatically improves SM prediction of key observables
  - M_W (28→11 MeV), $\sin^2\theta^l_{eff}$ ($2.3\times10^{-5} \to 1.0\times10^{-5}$), m_t (6.2→2.5 GeV)

- Improved accuracies set benchmark for new direct measurements!

- δM_W (indirect) = 11 MeV
  - Large contributions to δM_W (and $\delta\sin^2\theta^l_{eff}$) from top and unknown higher-order EW corrections

- δM_W (direct) = 15 MeV
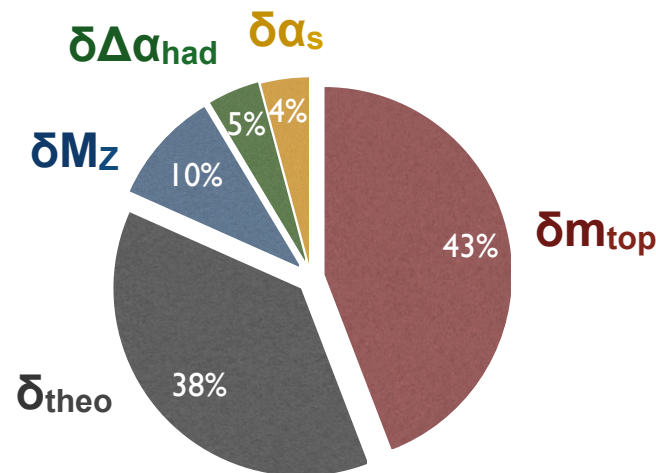
- Latest results always available at: http://cern.ch/Gfitter
  - Results in this presentation: EPJC 72, 2205 (2012)

G fitter

A Generic Fitter Project for HEP Model Testing

Backup

Goodness of Fit

- ■ Toy analysis with 20k pseudo experiments
  - • p-value = probability of getting $\chi^2_{min, toy}$ larger than χ^2_{min} from data
  - • i.e probability of incorrectly rejecting the SM as false = 0.07 ± 0.01 (theo)

A Gfitter package for Oblique Corrections

G fitter $^{B}_{SM}$
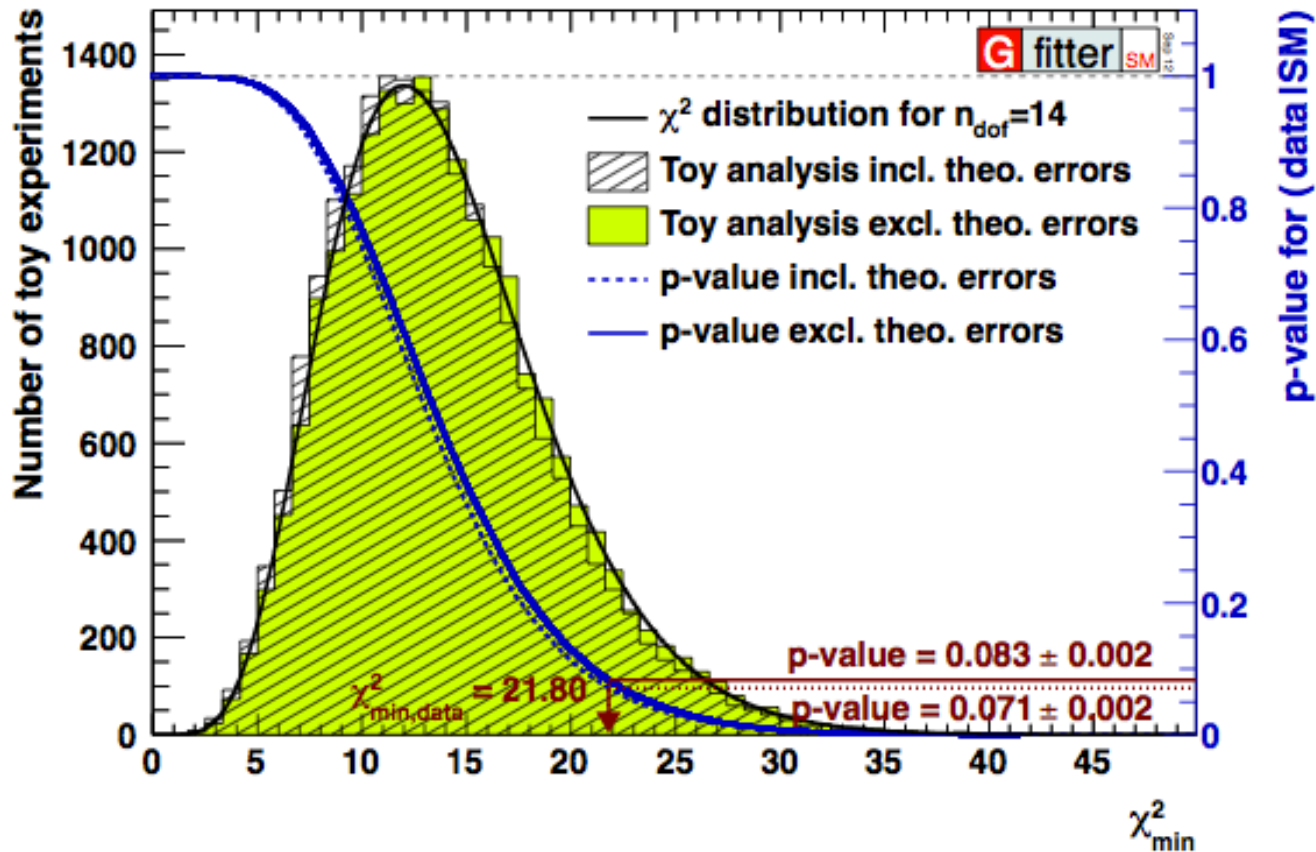
- **At low energies, BSM physics appears dominantly through vacuum polarization corrections**
  - Aka, "oblique corrections"

- **Oblique corrections reabsorbed into electroweak parameters**
  - $\Delta\rho$, $\Delta\kappa$, Δr parameters, appearing in: M_W^2, $\sin^2\theta_{eff}$, G_F, α, etc

- **Electroweak fit sensitive to BSM physics through oblique corrections**
  - In direct competition with sensitivity to Higgs loop corrections
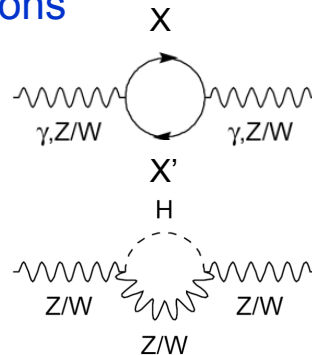
- **Oblique corrections from New Physics described through STU parametrization**
  [Peskin and Takeuchi, Phys. Rev. D46, 1 (1991)]

$$O_{meas} = O_{SM,REF}(m_H, m_t) + c_S S + c_T T + c_U U$$

- S : New Physics contributions to neutral currents
- T : Difference between neutral and charged current processes – sensitive to weak isospin violation
- U : (+S) New Physics contributions to charged currents. U only sensitive to W mass and width, usually very small in NP models (often: U=0)

- **Also implemented: correction to Z→bb coupling, extended parameters (VWX)**
  [Burgess et al., Phys. Lett. B326, 276 (1994)]
  [Burgess et al., Phys. Rev. D49, 6115 (1994)]

- The branching ratio R^0_b: partial decay width of $Z \to bb$ to $Z \to qq$

- Freitas et al: full 2-loop calculation of $Z \to bb$

- Contribution of same terms as in the calculation of $\sin^2\theta^{bb}_{eff}$
  $\to$ cross-check of two results found good agreement

- Two-loop corrections comparable to experimental uncertainty (6.6×10^{-4})

M_H [GeV]	1-loop EW and QCD correction to FSR $\mathcal{O}(\alpha) + \text{FSR}_{1-\text{loop}}$ $[10^{-3}]$	2-loop EW correction $\mathcal{O}(\alpha^2_{\text{ferm}})$ $[10^{-4}]$	2-loop EW and 2+3-loop QCD correction to FSR $\mathcal{O}(\alpha^2_{\text{ferm}}) + \text{FSR}_{>1-\text{loop}}$ $[10^{-4}]$	1+2-loop QCD correction to gauge boson self-energies $\mathcal{O}(\alpha\alpha_s, \alpha\alpha^2_s)$ $[10^{-4}]$
100	-3.632	-6.569	-9.333	-0.404
200	-3.651	-6.573	-9.332	-0.404
400	-3.675	-6.581	-9.331	-0.404

$\alpha_s(M_Z)$ from Z→hadrons

- Determination of α_s at N3LO.

- Most sensitive through total hadronic cross-section σ^0_{had} and partial leptonic width R^0_l

- Theory uncertainty obtained by scale variation, at per-mille level.
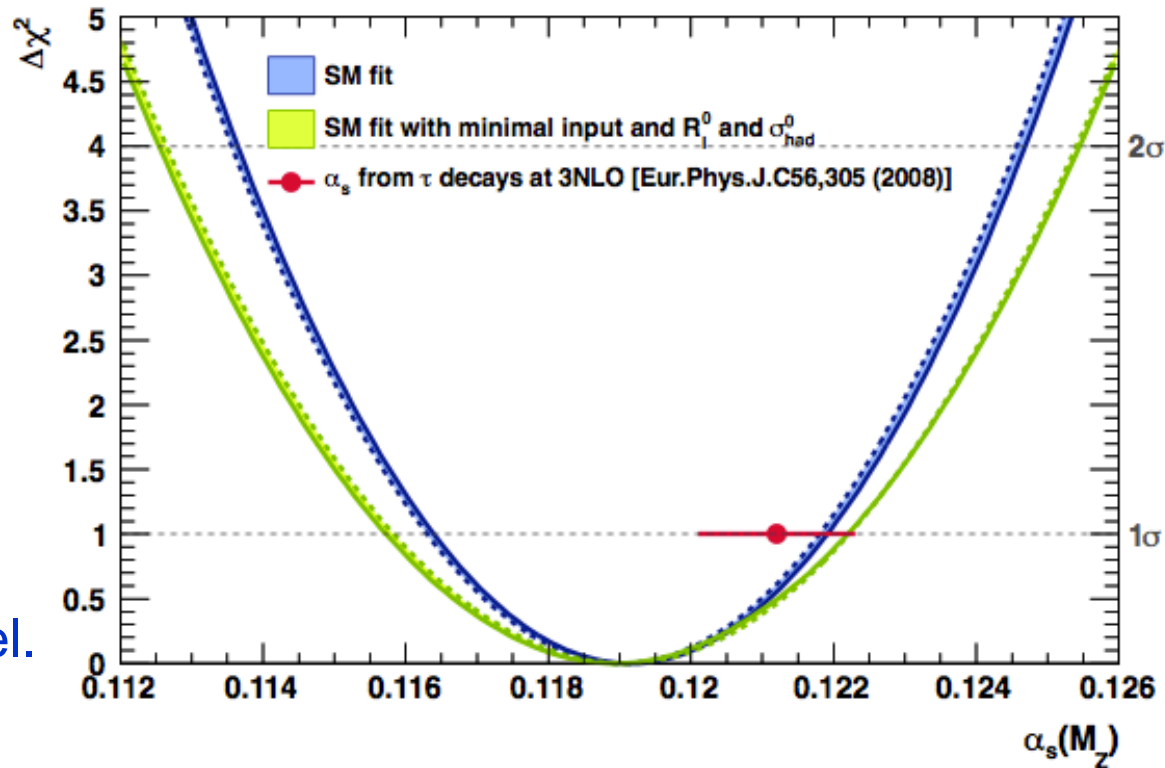
$$\alpha_s(M_Z) = 0.1191 \pm 0.0028\,(\text{exp.}) \pm 0.0001\,(\text{theo.})$$

- Good agreement with value from t decays, also at N3LO.

- Improvements in precision only expected with ILC/GigaZ

Higgs couplings in the EW fit

- In latest ATLAS H→γγ, 2.3σ deviation seen from SM μ (≡1.0)

- Interpret.: H→VV couplings scaled with c_V

From: Falkowski et al, arXiv:1303.1812

- Modified Higgs couplings can be constrained by EW fit through extended STU formalism.

- Result of c_V driven by limit on T parameter.
  - Tree-level relation: $\rho_0 = \dfrac{M_{W_0}^2}{M_{Z_0}^2 c_W^2} = 1 + \alpha T$

  - $\alpha T \approx \dfrac{3 g_Y^2}{32\pi^2}\left(c_V^2 - 1\right)\log(\Lambda/m_Z)$

  - Reminder: T = 0.05 ± 0.12 (Gfitter)

- EW-fit Falkowski et al: $c_V \simeq 1.08 \pm 0.07$
  - Blue dashed: c_V from μ's, black: comb. w/ EW

ATLAS Preliminary
Data 2012, √s = 8 TeV
∫Ldt = 20.7 fb⁻¹
H → γγ
m_H = 126.8 GeV

Signal strength

Falkowski et al, arXiv:1303.1812