Fits of EWPO in the SM with ILC/FCC-ee Precision

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First FCC-ee Mini-workshop on Precision Observables and Radiative Corrections
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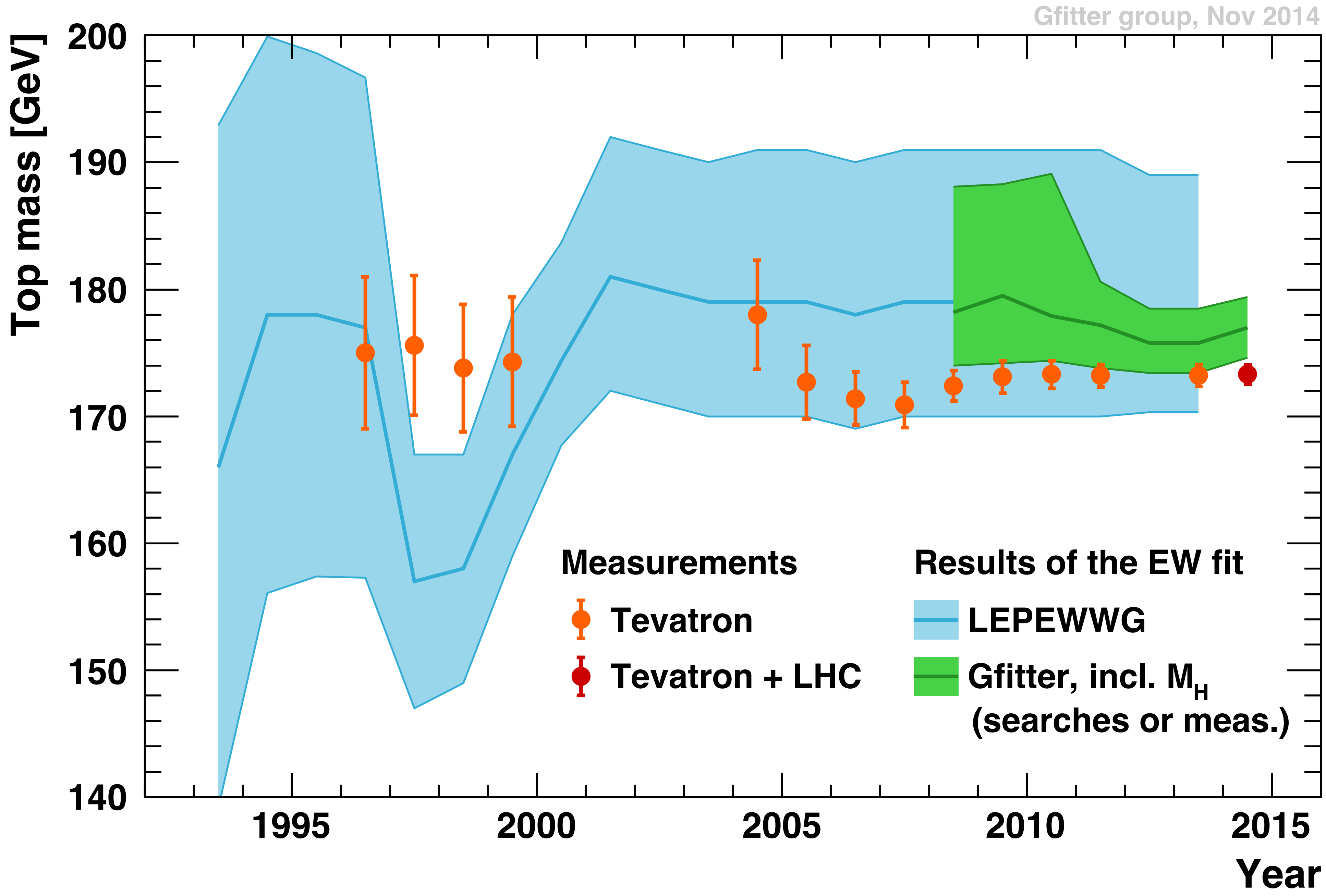
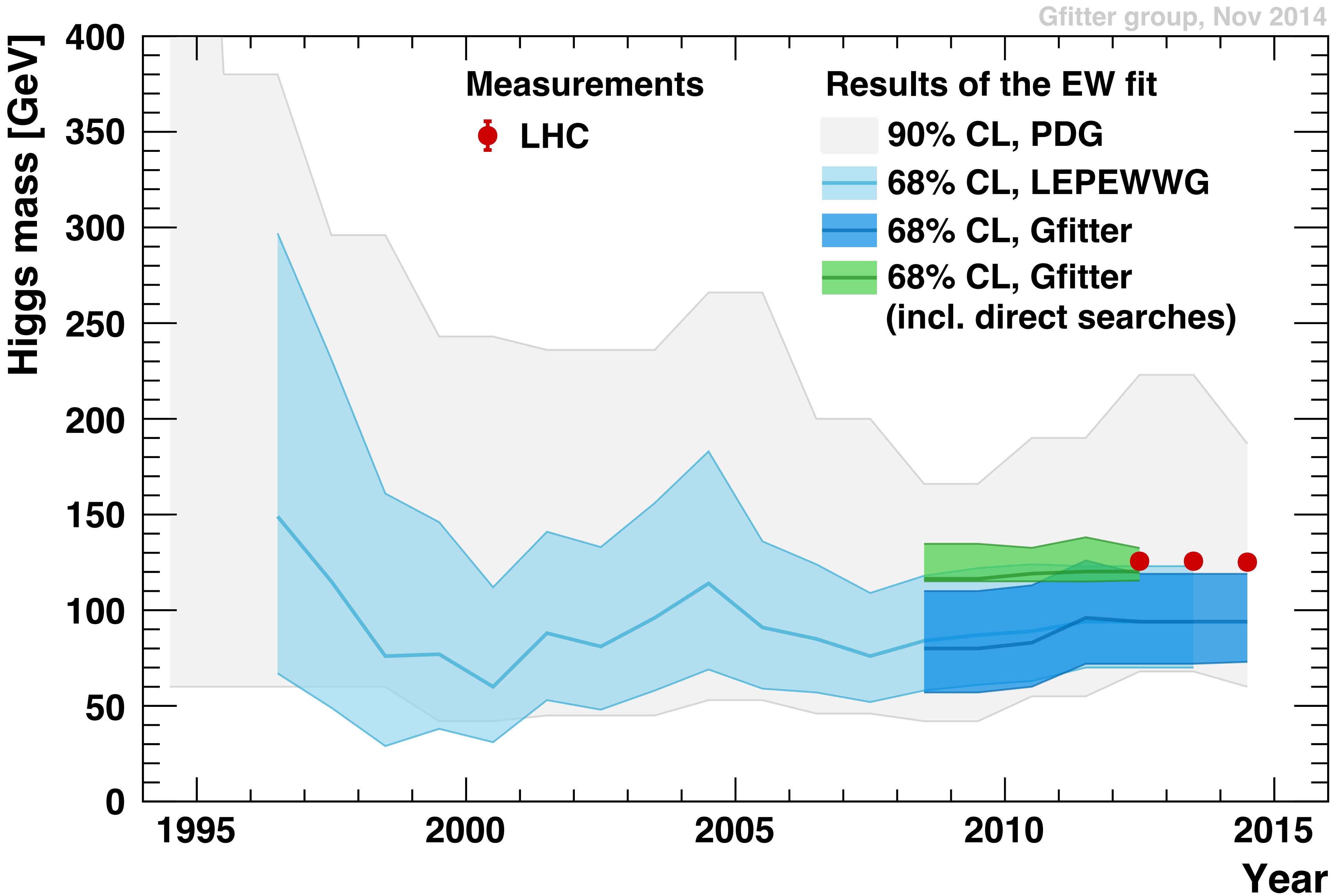
Prediction of Top Quark Mass



- m_t predictions from loop effects since 1990
- official LEPEWWG fit since 1993
- the fits have always been able to predict m_t correctly!

What precision is needed to see significant deviations between measurements and predictions?

Prediction of Higgs Mass



- M_H predictions from loop effects since the discovery of the top quark 1995
- weaker constraints than for m_t because of logarithmic dependence
- still, the fits have always predicted M_H correctly!

Again: what precision should we strive for? What are the major challenges?

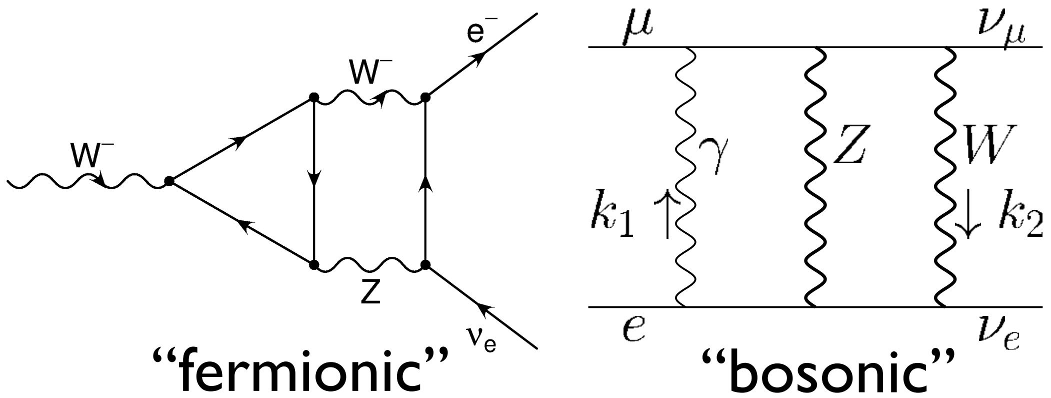
Present: Experimental Input

Fit is overconstrained

- all free parameters measured
  - most input from e^+e^- colliders
    - M_Z : $2\cdot 10^{-5}$
  - but crucial input from hadron colliders:
    - m_t : $4\cdot 10^{-3}$
    - M_H : $2\cdot 10^{-3}$
    - M_W : $2\cdot 10^{-4}$
  - remarkable experimental precision (<1%)
- require precision calculations!

M_H [GeV]$^{(\circ)}$	125.14 ± 0.24	LHC
M_W [GeV]	80.385 ± 0.015	Tev.
Γ_W [GeV]	2.085 ± 0.042	Tev.
M_Z [GeV]	91.1875 ± 0.0021	LEP
Γ_Z [GeV]	2.4952 ± 0.0023	LEP
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	LEP
R^0_ℓ	20.767 ± 0.025	LEP
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	LEP
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	LEP, SLD
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	LEP
A_c	0.670 ± 0.027	SLD
A_b	0.923 ± 0.020	SLD
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	LEP
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	LEP
R^0_c	0.1721 ± 0.0030	LEP, SLD
R^0_b	0.21629 ± 0.00066	LEP, SLD
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]	173.34 ± 0.76	Tev.+LHC
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	2757 ± 10	

Calculations



All observables calculated at 2-loop level

- M_W : full EW one- and two-loop calculation of fermionic and bosonic contributions
  [Awramik et al., PRD 69, 053006 (2004), PRL 89, 241801 (2002)]
  + 4-loop QCD correction [Chetyrkin et al., PRL 97, 102003 (2006)]
- $\sin^2\theta^l_{eff}$: same order as M_W, calculations for leptons and all quark flavours
  [Awramik et al, PRL 93, 201805 (2004), JHEP 11, 048 (2006), Nucl. Phys. B813, 174 (2009)]
- partial widths Γ_f : fermionic corrections known to two-loop level for all flavours (includes predictions for σ^0_{had}) [Freitas, JHEP04, 070 (2014)]
- Radiator functions: QCD corrections at N^{3}LO [Baikov et al., PRL 108, 222003 (2012)]
- Γ_W : only one-loop EW corrections available, negligible impact on fit
  [Cho et al, JHEP 1111, 068 (2011)]
- all calculations include one- and two-loop QCD corrections and leading terms of higher order corrections

All EWPOs calculated at two-loop level or better

Theoretical Uncertainties

Estimation

- assume that perturbative expansion follows a geometric series ($a_n = a\, r^n$) :

  for example: $\mathcal{O}(\alpha^2\alpha_s) = \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}\,\mathcal{O}(\alpha\alpha_s)$

- other methods (e.g. scale variation) not always feasible
  - but give ~similar results
- theoretical uncertainties smaller by a factor of 3-6 than measurements
  - for the first time, reasonable estimate for all observables

important

Observable	Exp. error	Theo. error
M_W	15 MeV	4 MeV
$\sin^2\theta^l_{\rm eff}$	$1.6 \cdot 10^{-4}$	$0.5 \cdot 10^{-4}$
Γ_Z	2.3 MeV	0.5 MeV
$\sigma^0_{\rm had} = \sigma[e^+e^- \to Z \to \text{had.}]$	37 pb	6 pb
$R^0_b = \Gamma[Z \to b\bar{b}]/\Gamma[Z \to \text{had.}]$	$6.6 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$
m_t	0.76 GeV	0.5 GeV

new in fit

- important missing higher order terms:
  - $O(\alpha^3)$, $O(\alpha^2\alpha_s)$, $O(\alpha\alpha_s^2)$, $O(\alpha^2_{bos})$ (in some cases), $O(\alpha_s^5)$ (rad. functions)

Future Improvements

Parameter	Present	LHC	ILC/GigaZ	
M_H [GeV]	0.2 →	< 0.1	< 0.1	
M_W [MeV]	15 →	8 →	5	WW threshold
M_Z [MeV]	2.1	2.1	2.1	
m_t [GeV]	0.8 →	0.6 →	0.1	$t\bar{t}$ threshold scan
$\sin^2\theta^{\ell}_{\rm eff}$ $[10^{-5}]$	16	16 →	1.3	$\delta A^{0,f}_{LR}$: $10^{-3} \rightarrow 10^{-4}$
$\Delta\alpha^5_{\rm had}(M_Z^2)$ $[10^{-5}]$	10 →	4	4	low energy data, better α_s
R^0_l $[10^{-3}]$	25	25 →	4	high statistics on Z-pole
κ_V ($\lambda = 3\,$TeV)	0.05 →	0.03 →	0.01	direct measurement of BRs

LHC = LHC with 300 fb^{-1}
ILC/GigaZ = future e^+e^- collider, option to run on Z-pole (w polarized beams)

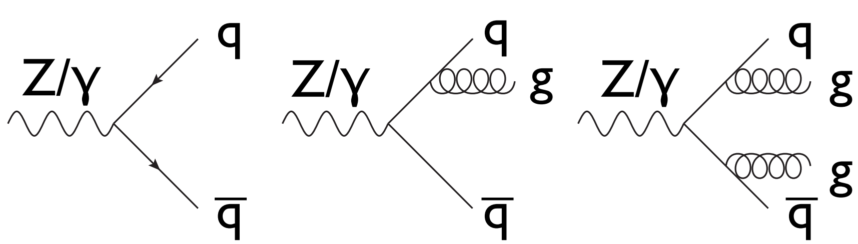
- theoretical uncertainties reduced by a factor of 4 (esp. M_W and $\sin^2\theta^l_{\rm eff}$)
  - implies three-loop calculations!
  - exception: $\delta_{\rm theo}\, m_t$ (LHC) = 0.25 GeV (factor 2)
- central values of input measurements adjusted to M_H = 125 GeV

[Baak et al, arXiv:1310.6708]

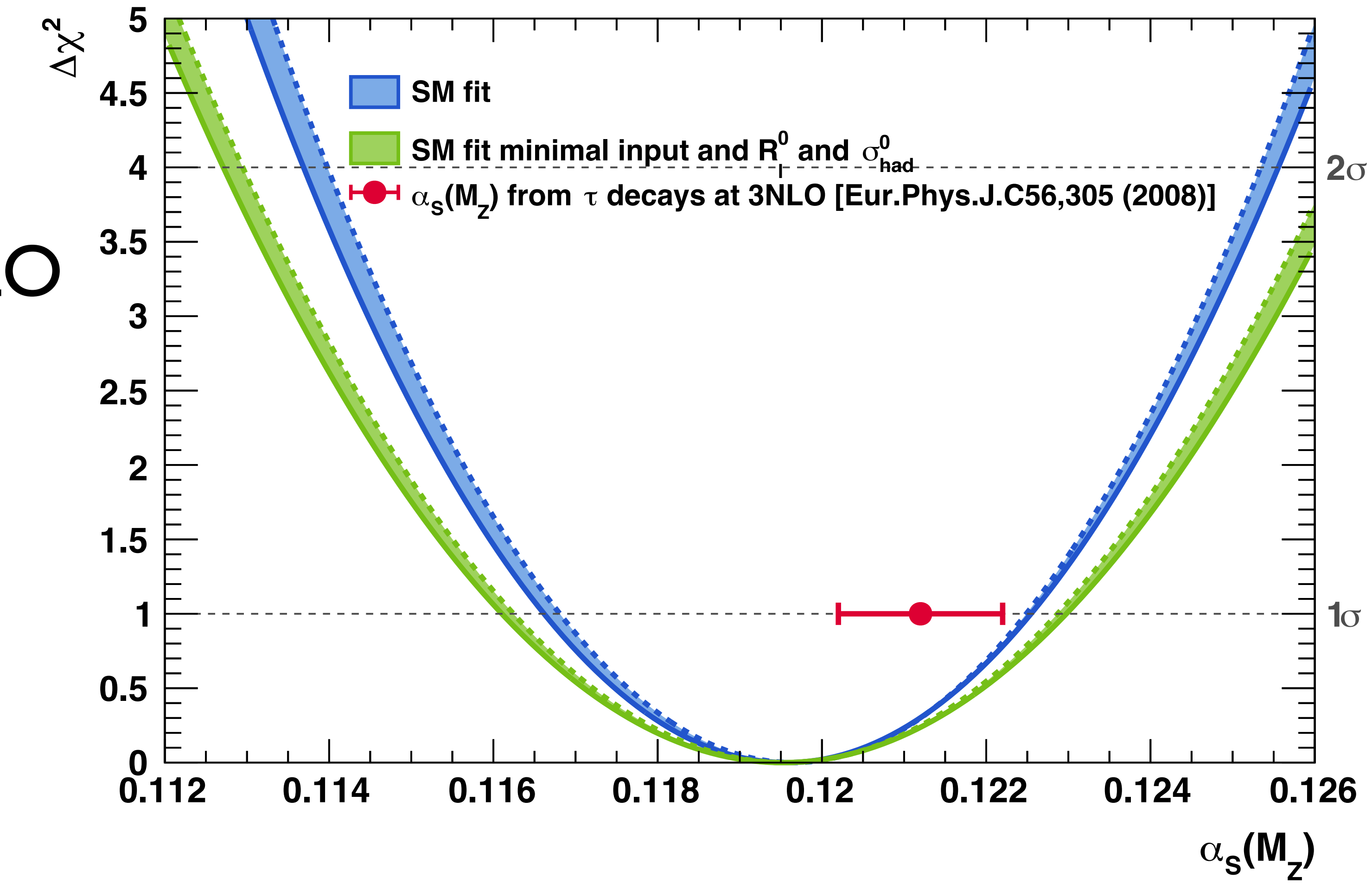
Present: the Strong Coupling $\alpha_s(M_Z)$

$\Delta\chi^2$ profile vs $\alpha_s(M_Z)$

- determination of α_s at full NNLO and partial NNNLO
- also shown: minimal input with two most sensitive measurements: R_l, σ^0_{had}



- M_H has no (visible) impact



$$\alpha_s(M_Z^2) = 0.1196 \pm 0.0028_{\rm exp} \pm 0.0006_{\delta_{\rm theo}\mathcal{R}_{V,A}} \pm 0.0006_{\delta_{\rm theo}\Gamma_i} \pm 0.0002_{\delta_{\rm theo}\sigma^0_{\rm had}}$$
$$= 0.1196 \pm 0.0030_{\rm tot}$$

More accurate estimation of theo. uncertainties (previously: δ_{theo} = 0.0001 from scale variations)

dominated by exp. uncertainty

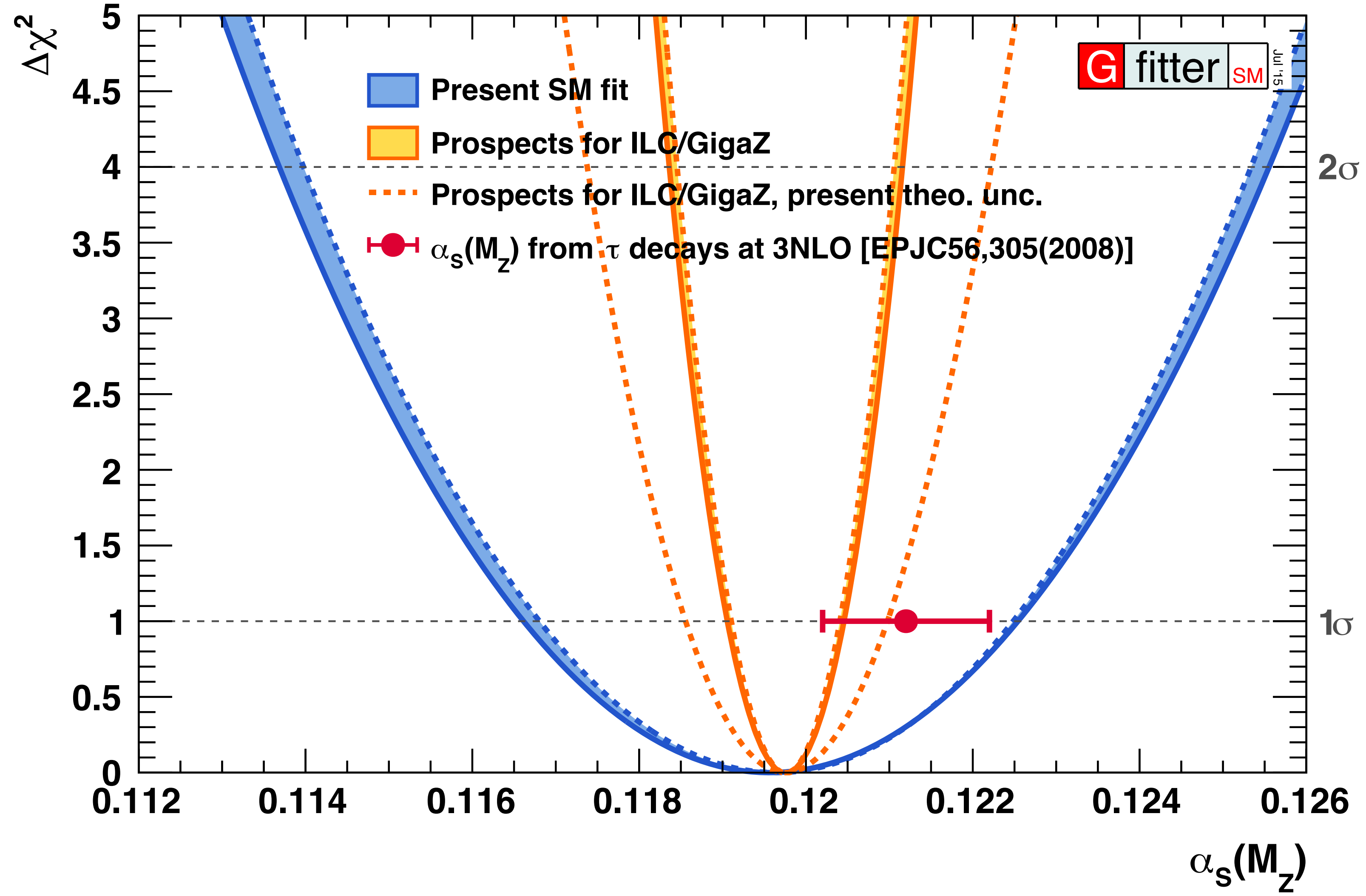
Future: the Strong Coupling $\alpha_s(M_Z)$

LHC-300 Scenario

- no improvement

ILC Scenario

- improvement of factor 4 or better possible
  - needs improvement from theory
  - present uncertainties: factor of 2.5 only



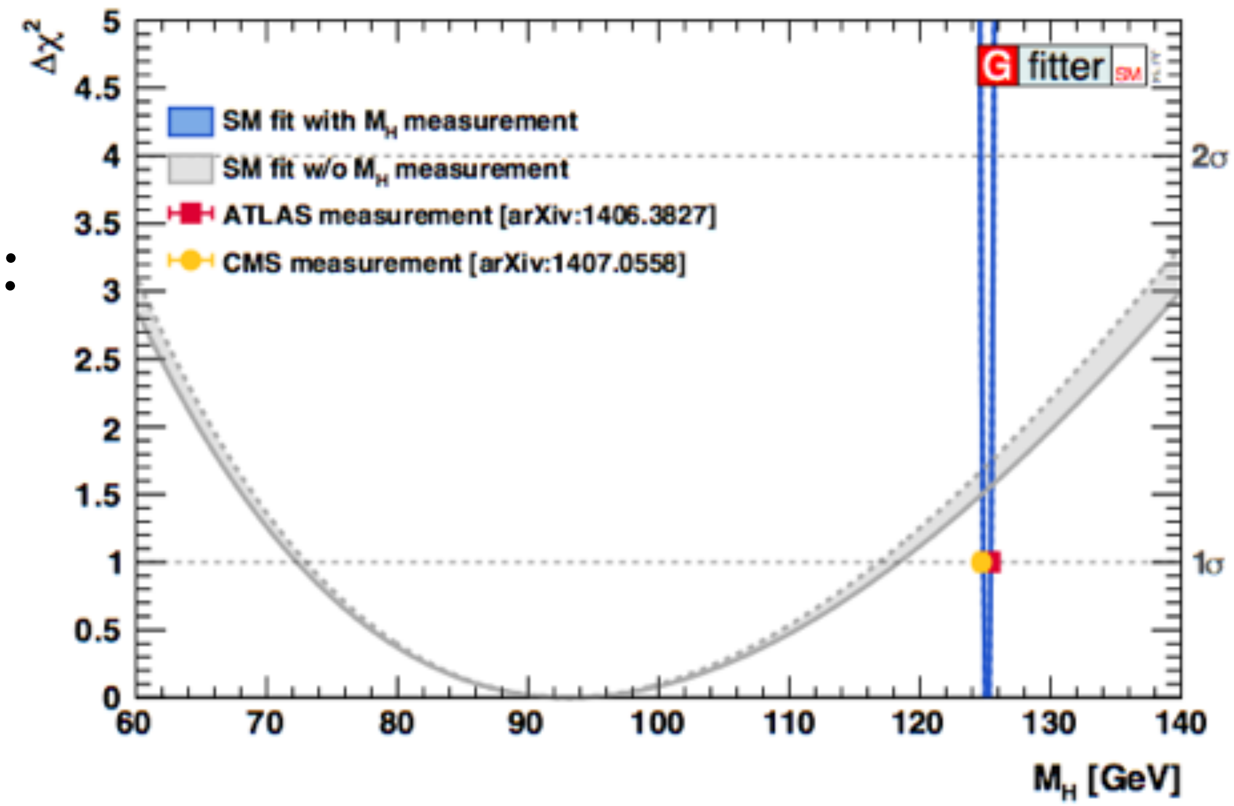
Fit Results:

$$\delta\alpha_s = (6.5_{\text{exp}} \oplus 2.5_{\delta_{\text{theo}}\Gamma_i} \oplus 2.3_{\delta_{\text{theo}}\mathcal{R}_{V,A}}) \cdot 10^{-4}$$

$$\delta\alpha_s = (7.0_{\text{tot}}) \cdot 10^{-4}$$ (present theory uncertainty: $12.2 \cdot 10^{-4}$)

Promises most precise measurement of $\alpha_s(M_Z)$

Present Results: Higgs

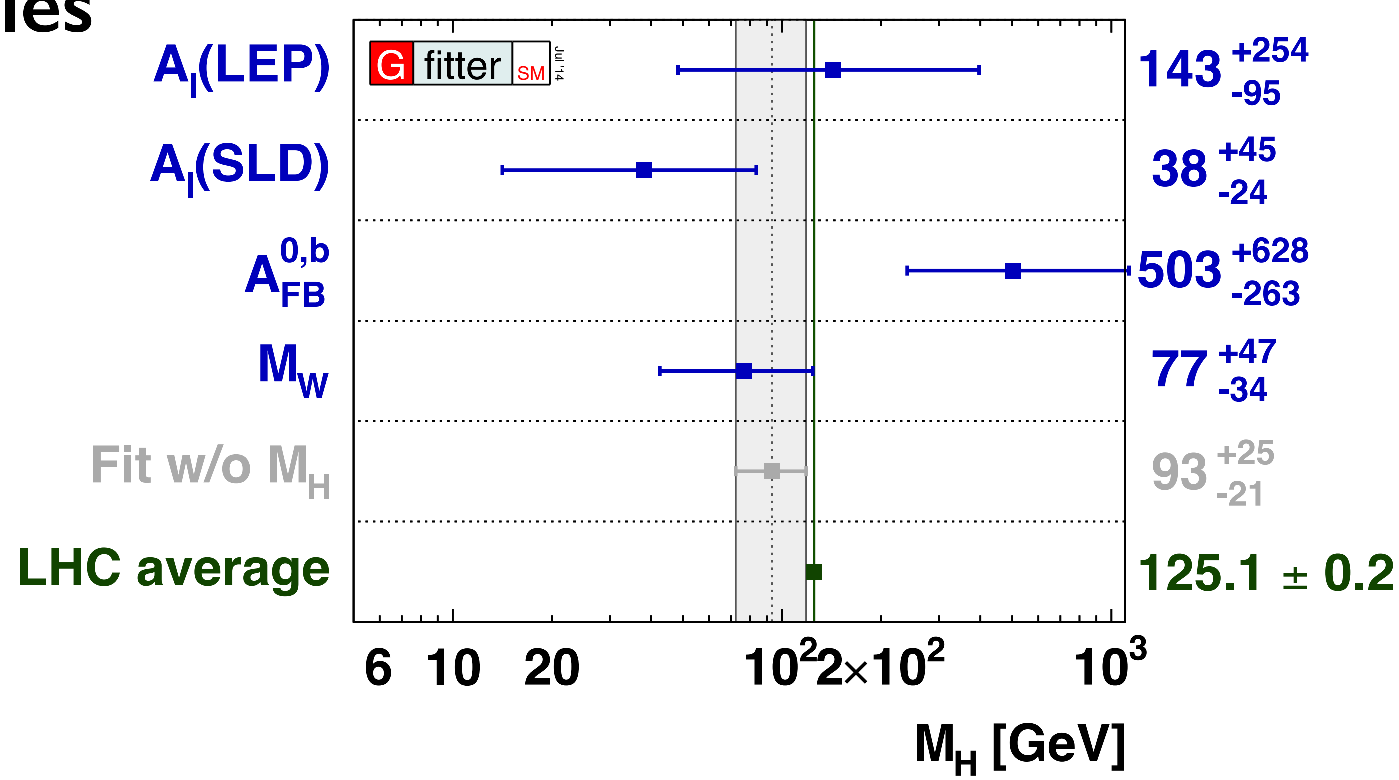


$\Delta\chi^2$ profile vs M_H

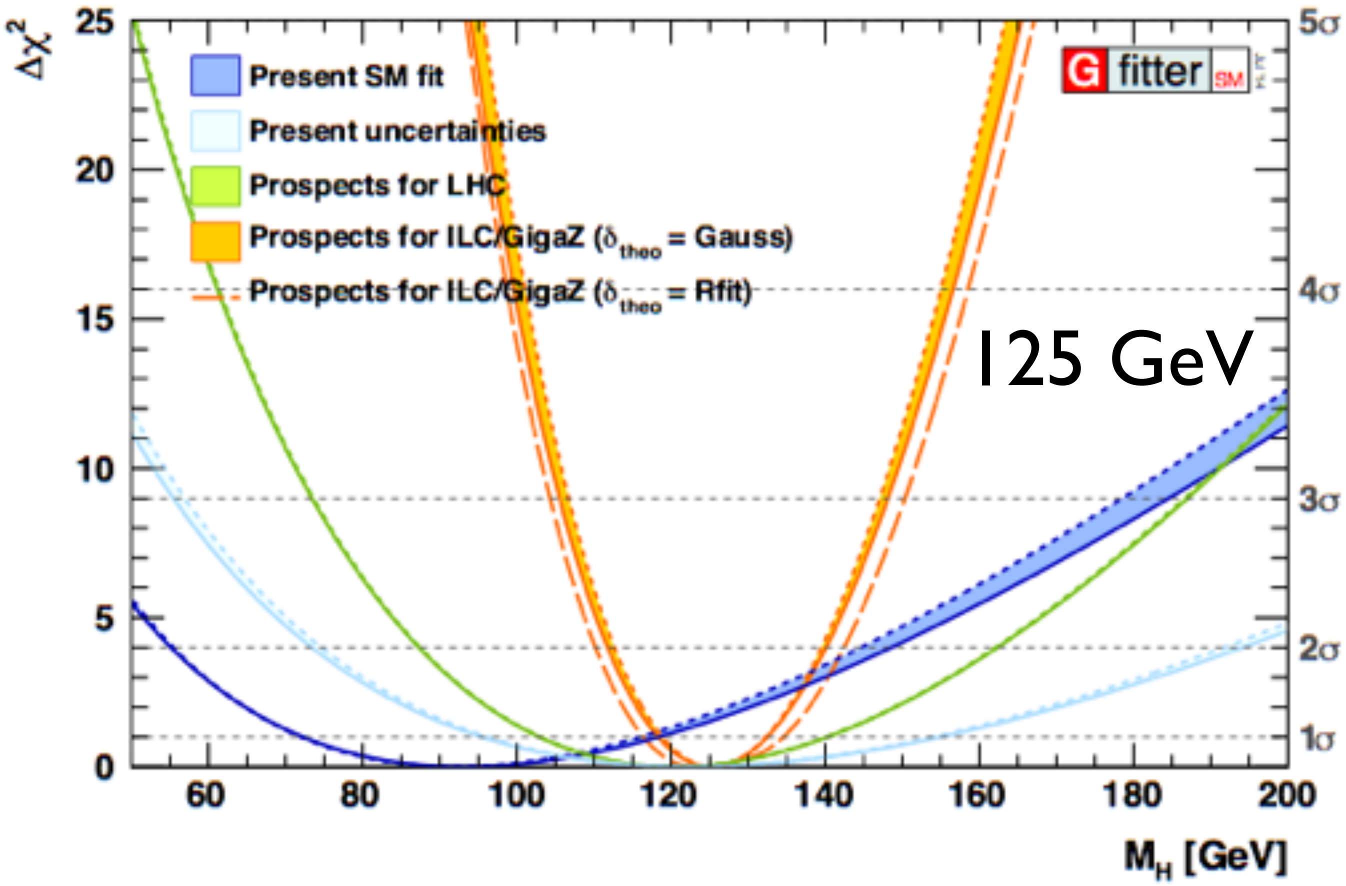
- grey band: fit without M_H measurement :
  - $M_H = 93^{+25}_{-21}$ GeV
  - consistent with measurement at **1.3σ**
- blue line: full SM fit

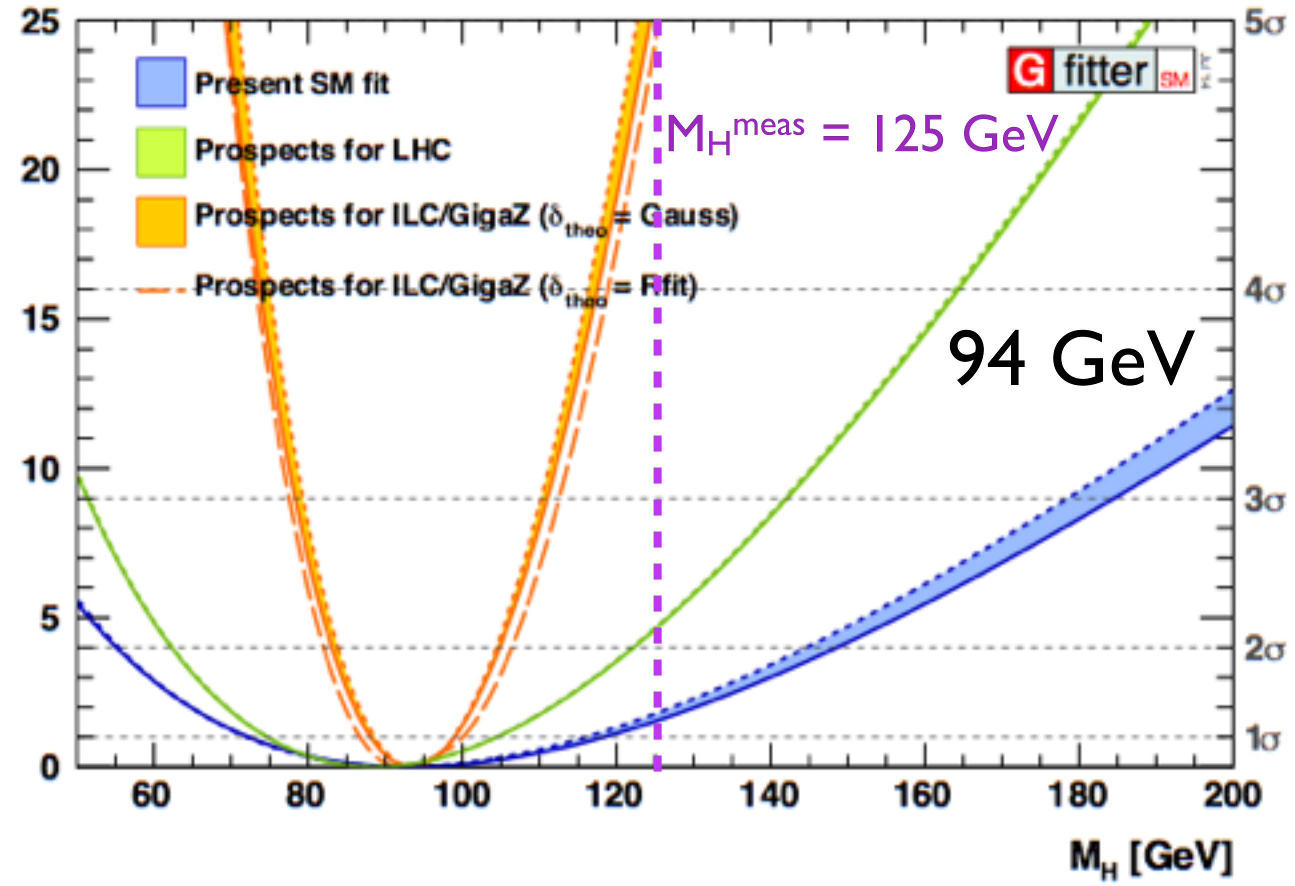
Impact of most sensitive observables

- determination of M_H, removing all sensitive observables except the given one
- known tension (3σ) between A_l(SLD), $A^{0,b}_{FB}$, and M_W clearly visible



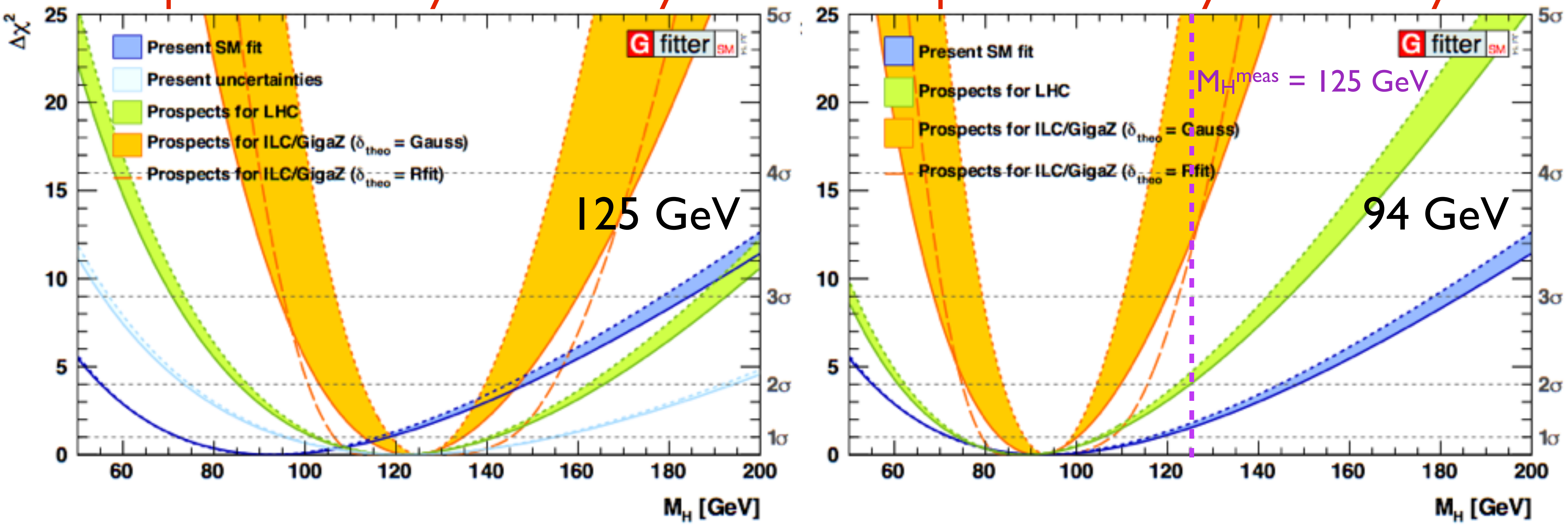
Future: Higgs Mass





- Logarithmic dependency on M_H → cannot compete with direct M_H meas.
  - no theory uncertainty: $M_H = 125 \pm 7$ GeV
  - future theory uncertainty (Rfit): $M_H = 125^{+10}_{-9}$ GeV
  - present day theory uncertainty: $M_H = 125^{+20}_{-17}$ GeV
- If EWPO central values unchanged (94 GeV), ~5σ discrepancy with measured Higgs mass

Future: Higgs Mass



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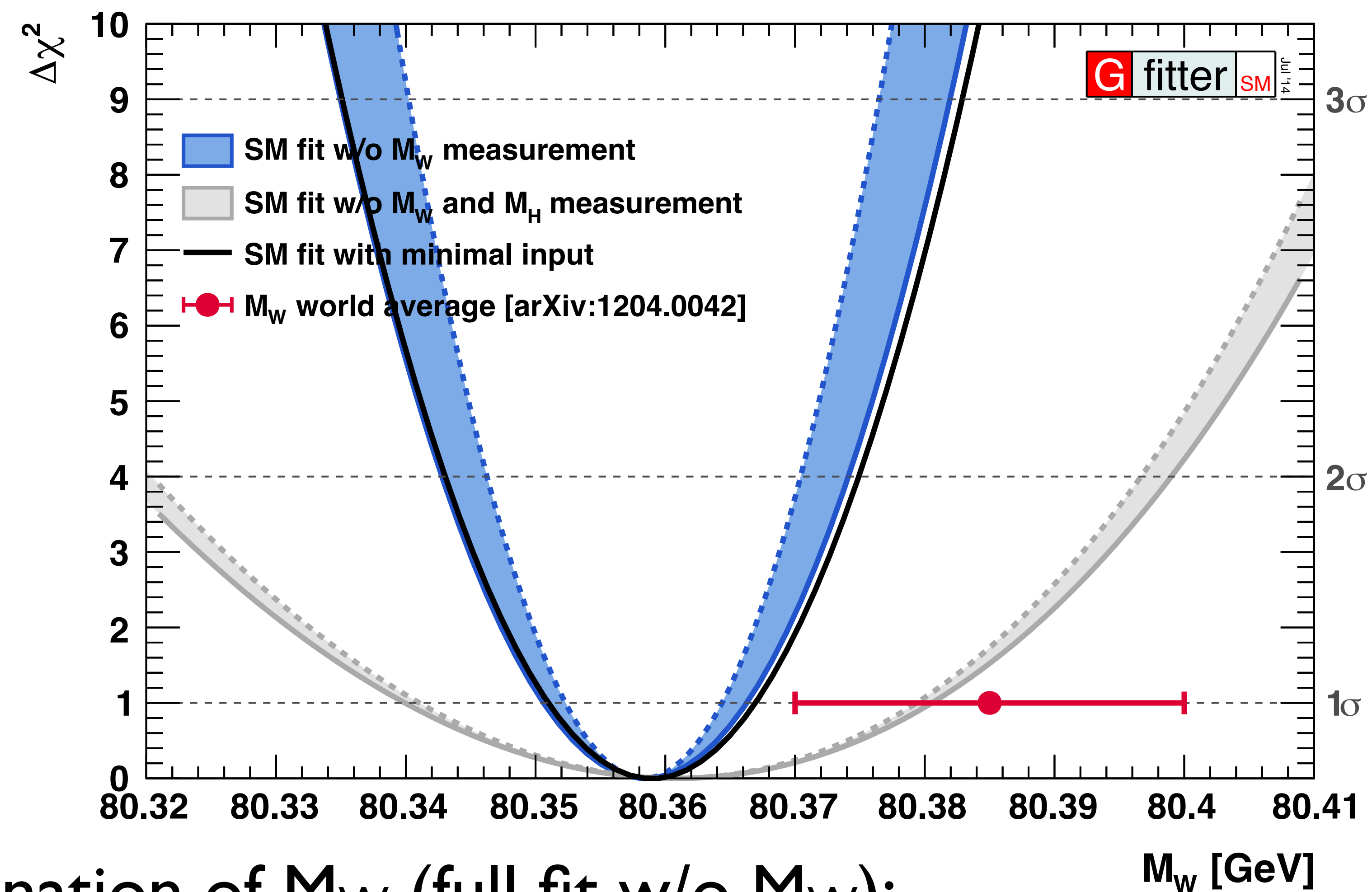
- If EWPO central values unchanged (94 GeV), ~5σ discrepancy with measured Higgs mass compromised by present theory uncertainty!

Present Results: M_W



$\Delta\chi^2$ profile vs M_W

- also shown: SM fit with minimal input: M_Z, G_F, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, M_H, and fermion masses
  - good consistency
- M_H measurement allows for precise constraint on M_W
  - agreement at **1.4σ**
- fit result for indirect determination of M_W (full fit w/o M_W):

$$\begin{aligned} M_W &= 80.3584 \pm 0.0046_{m_t} \pm 0.0030_{\delta_{\text{theo}} m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\text{had}}} \\ &\quad \pm 0.0020_{\alpha_S} \pm 0.0001_{M_H} \pm 0.0040_{\delta_{\text{theo}} M_W} \text{ GeV}, \\ &= 80.358 \pm 0.008_{\text{tot}} \text{ GeV} \end{aligned}$$

more precise than direct measurement (15 MeV)

Present Results: M_W

$\Delta\chi^2$ profile vs M_W

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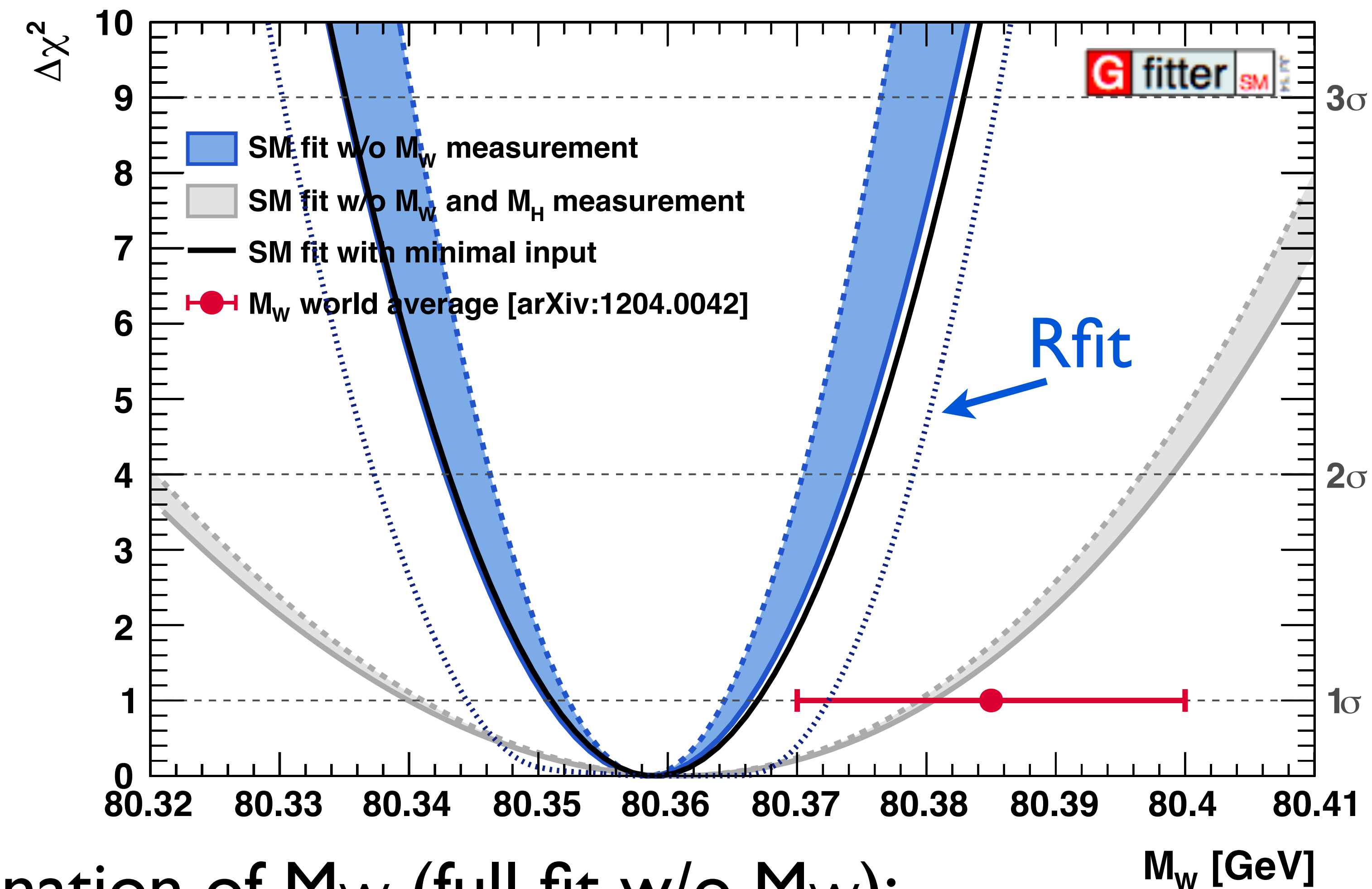

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$$= 80.358 \pm 0.008_{\rm tot}\ \text{GeV} \quad \text{(Rfit: ±13 MeV)}$$

more precise than direct measurement (15 MeV)

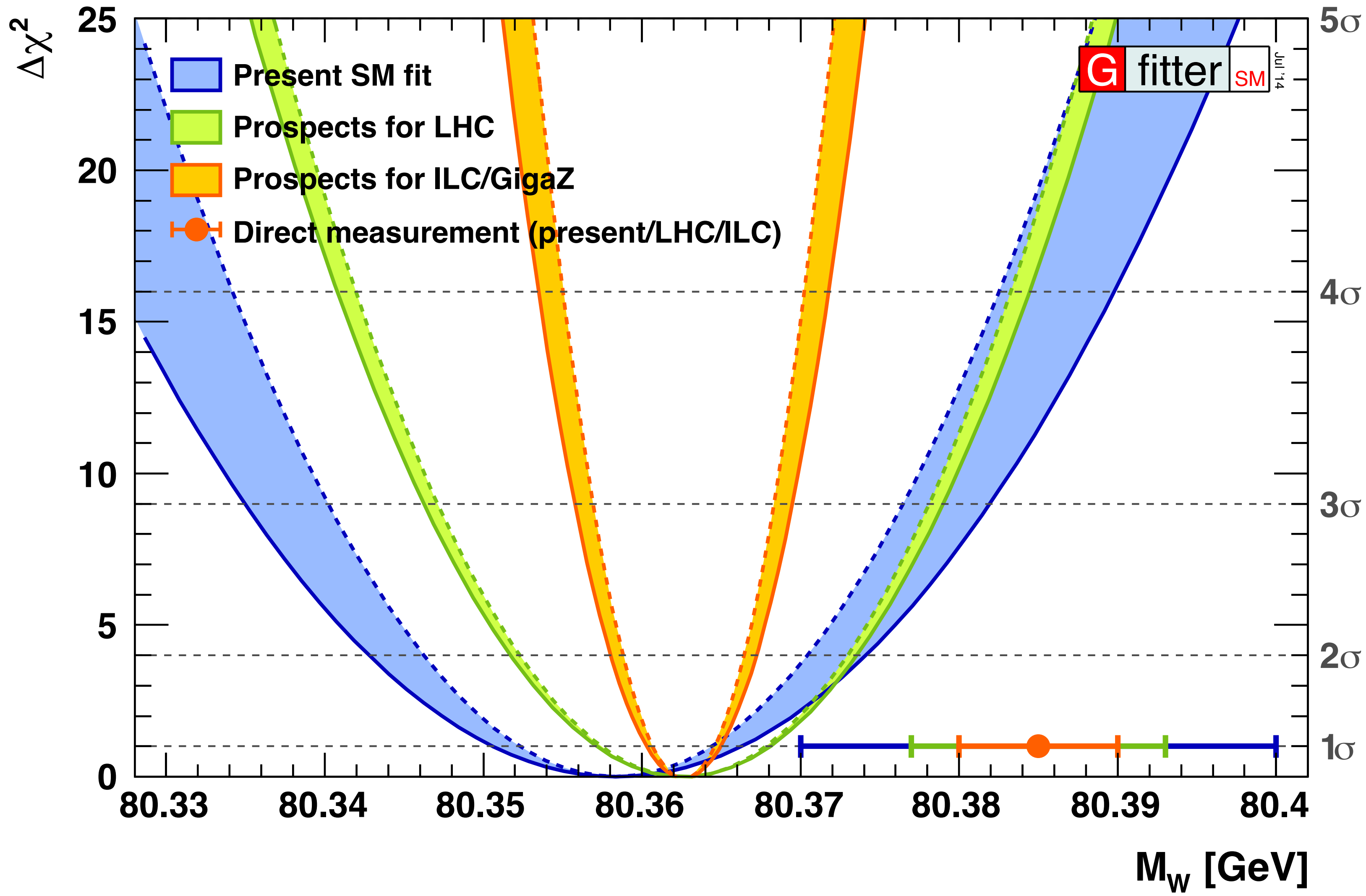
Future: M_W

LHC-300 Scenario

- moderate improvement (~30%) of indirect constraint
  - theoretical uncertainties already important

ILC Scenario

- improvement of factor 3 possible, similar to direct measurement



Fit Results:

$$\delta M_W = 1.7_{M_Z} \oplus 0.1_{m_t} \oplus 1.2_{\sin^2\theta^f_{\mathrm{eff}}} \oplus 0.6_{\Delta\alpha_{\mathrm{had}}} \oplus 0.3_{\alpha_s}\ \mathrm{MeV}$$

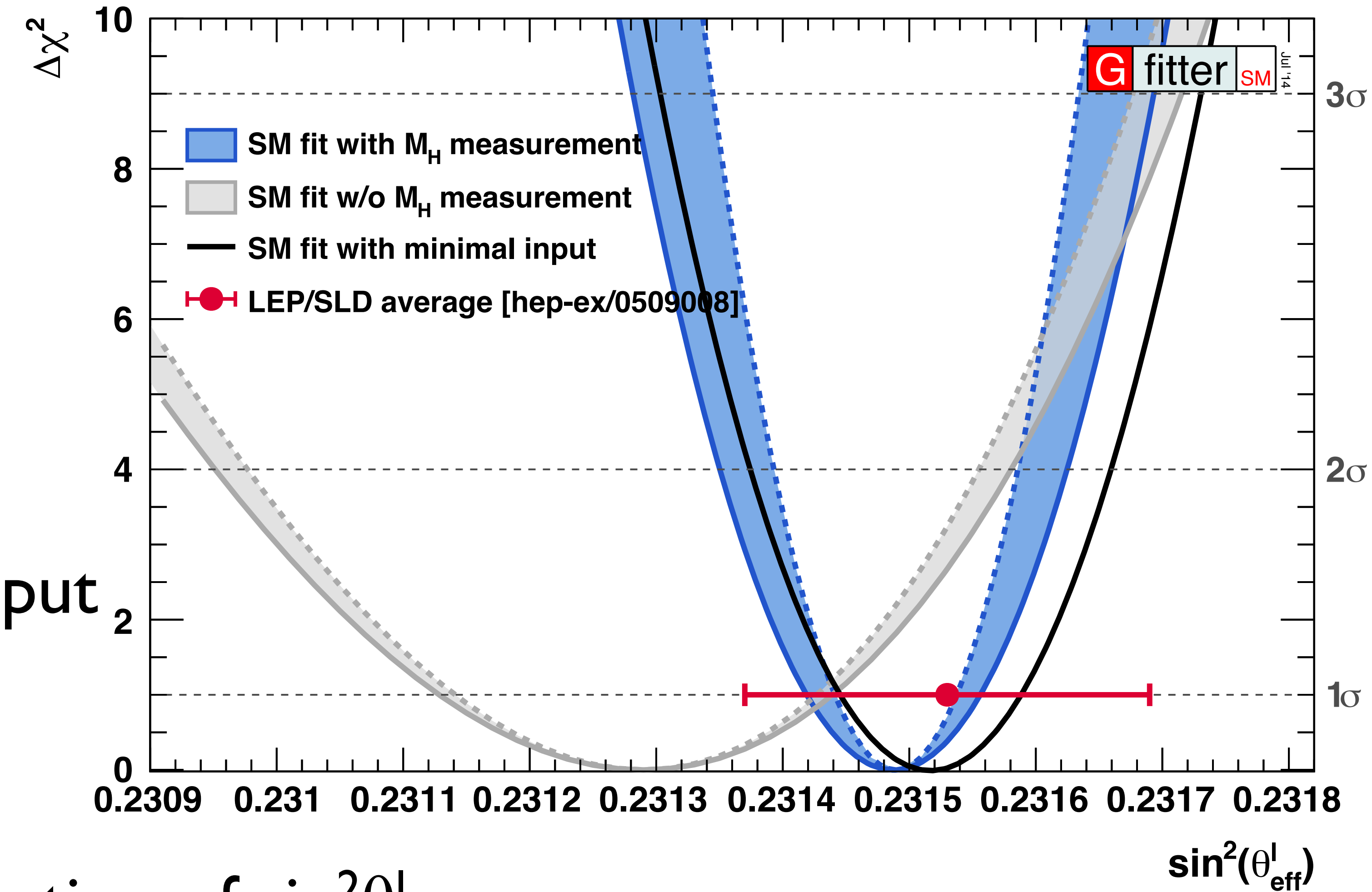
$$\delta M_W = 1.3_{\mathrm{theo}} \oplus 1.9_{\mathrm{exp}}\ \mathrm{MeV} = 2.3_{\mathrm{tot}}\ \mathrm{MeV}$$

Measurement uncertainty for ILC: 5 MeV

Present: Effective Weak Mixing Angle

$\Delta\chi^2$ profile vs $\sin^2\theta^l_{eff}$

- all measurements directly sensitive to $\sin^2\theta^l_{eff}$ removed from fit (asymmetries, partial widths)
  - good agreement with min input
- M_H measurement allows for precise constraint



- fit result for indirect determination of $\sin^2\theta^l_{eff}$:

$$\begin{aligned}\sin^2\theta^\ell_{\rm eff} &= 0.231488 \pm 0.000024_{m_t} \pm 0.000016_{\delta_{\rm theo} m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \\ &\quad \pm 0.000010_{\alpha_S} \pm 0.000001_{M_H} \pm 0.000047_{\delta_{\rm theo}\sin^2\theta^f_{\rm eff}} \\ &= 0.23149 \pm 0.00007_{\rm tot}\end{aligned}$$

more precise than determination from LEP/SLD (1.6×10^{-4})

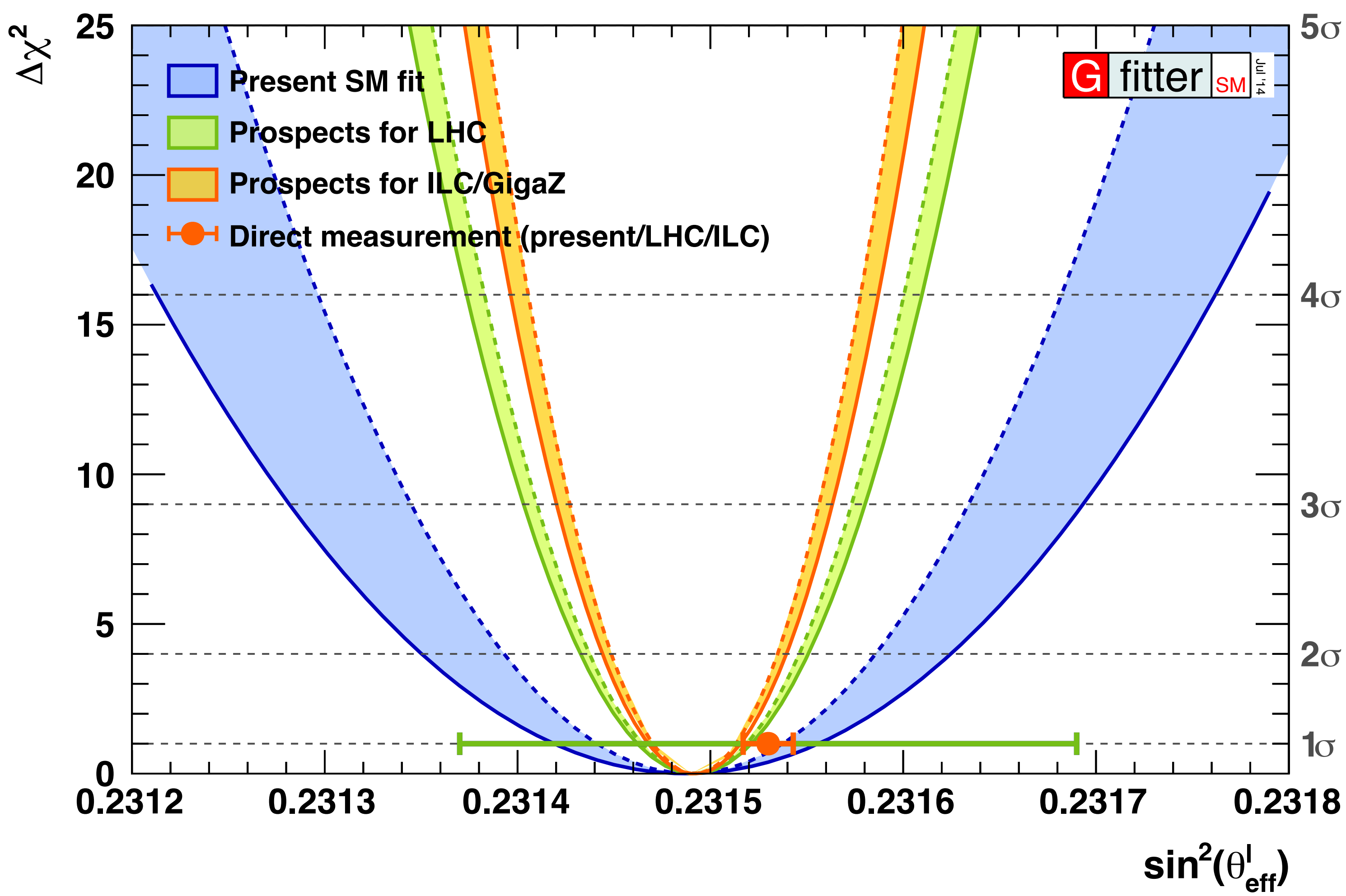
Future: Effective Weak Mixing Angle

LHC-300 Scenario

- large improvement of indirect constraint
  - compromised by today's theoretical uncertainties

ILC Scenario

- Indirect constraint and direct measurement comparable precision



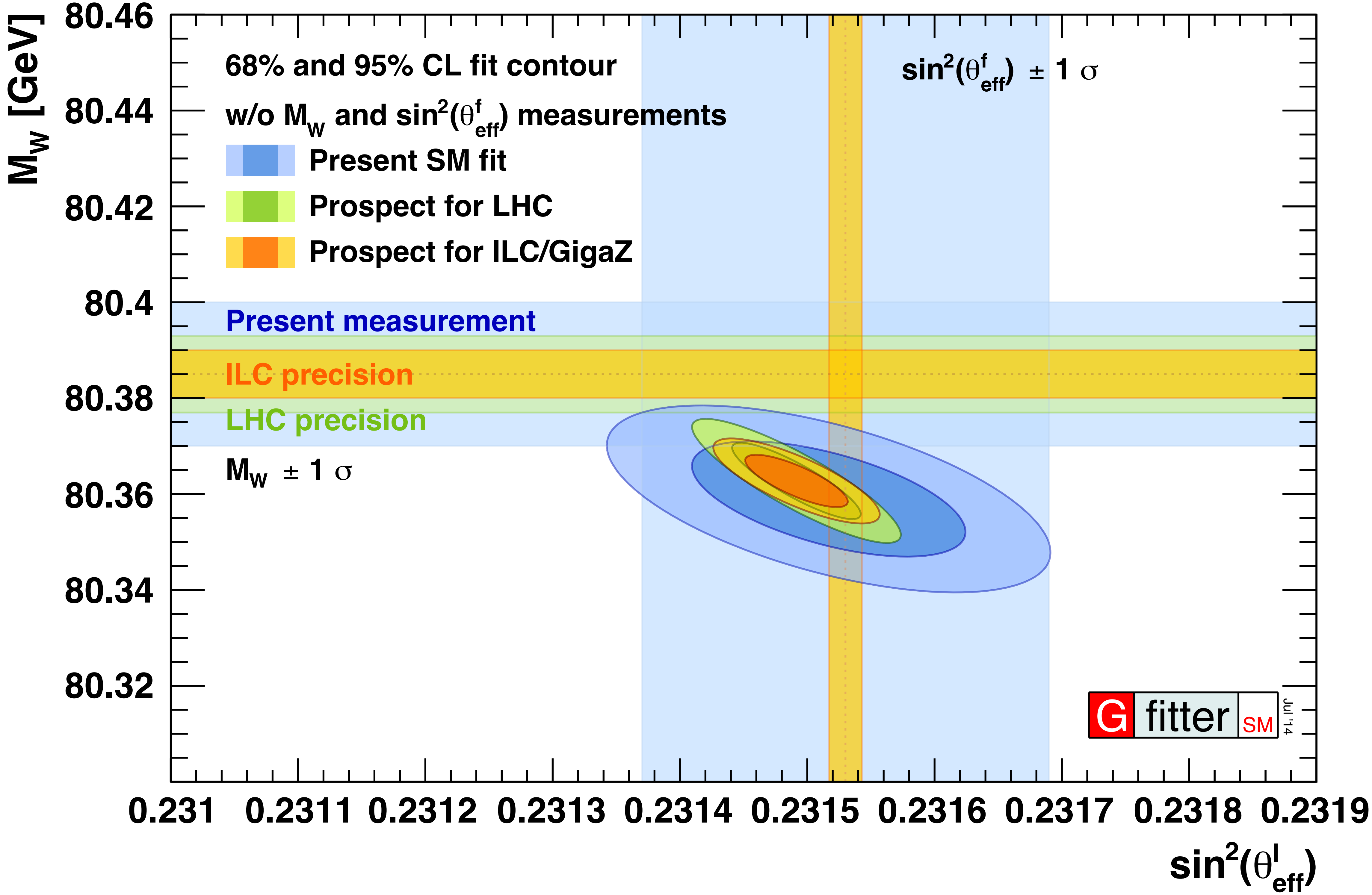
Fit Results:

$$\delta \sin^2 \theta^f_{\rm eff} = (1.7_{M_W} \oplus 1.2_{M_Z} \oplus 0.1_{m_t} \oplus 1.5_{\Delta\alpha_{\rm had}} \oplus 0.1_{\alpha_s}) \cdot 10^{-5}$$

$$\delta \sin^2 \theta^f_{\rm eff} = (1.0_{\rm theo} \oplus 2.0_{\rm exp}) \cdot 10^{-5} = (2.3_{\rm tot}) \cdot 10^{-5}$$

Measurement uncertainty for ILC: $1.3 \cdot 10^{-5}$

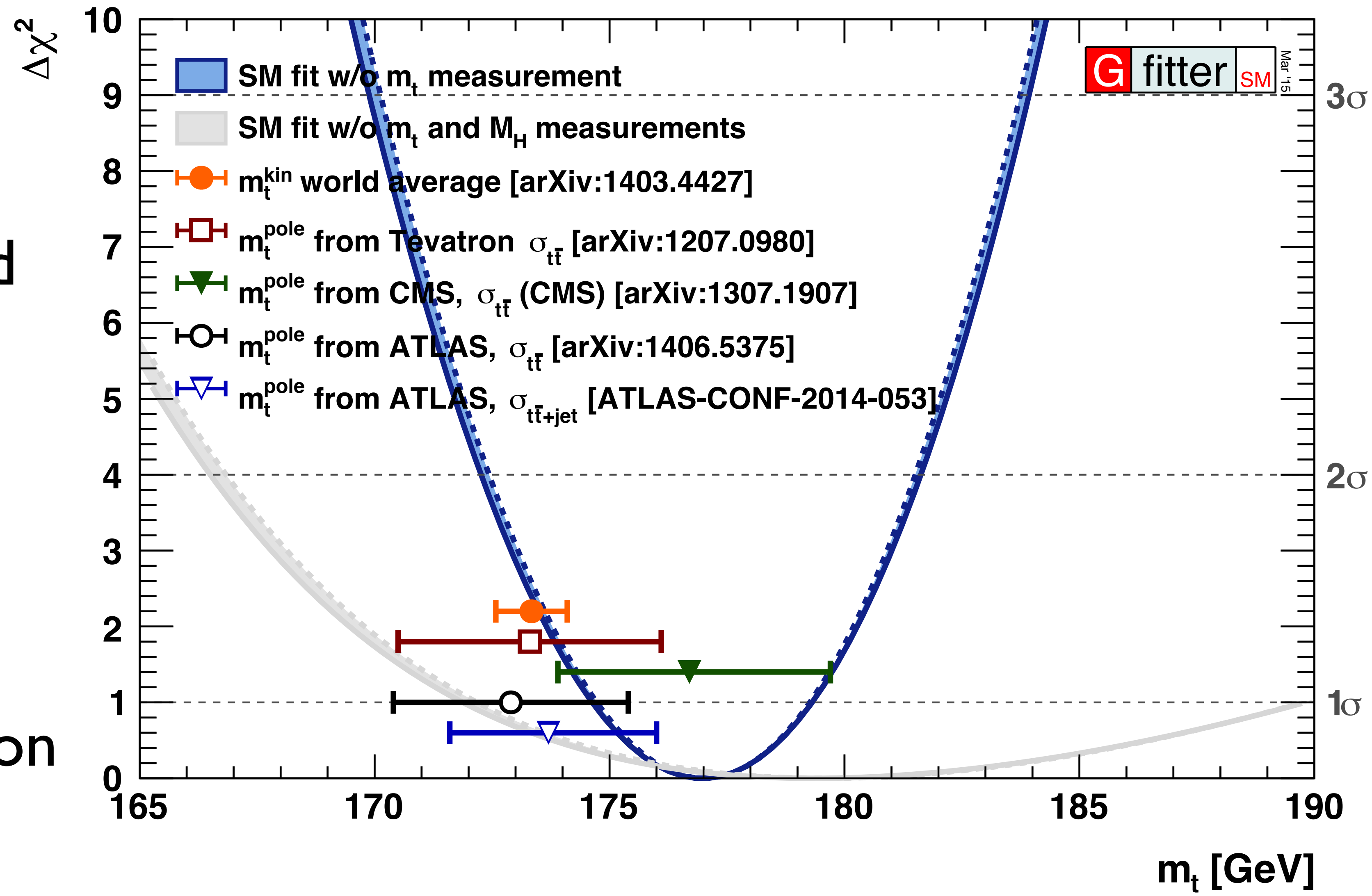
Future: M_W and $\sin^2\theta^l_{eff}$



Present: Top Quark Mass

$\Delta\chi^2$ profile vs m_t

- determination of m_t from Z-pole data (fully obtained from rad. corrections $\sim m_t^2$)
- alternative to direct measurements
- M_H allows for significantly more precise determination of m_t

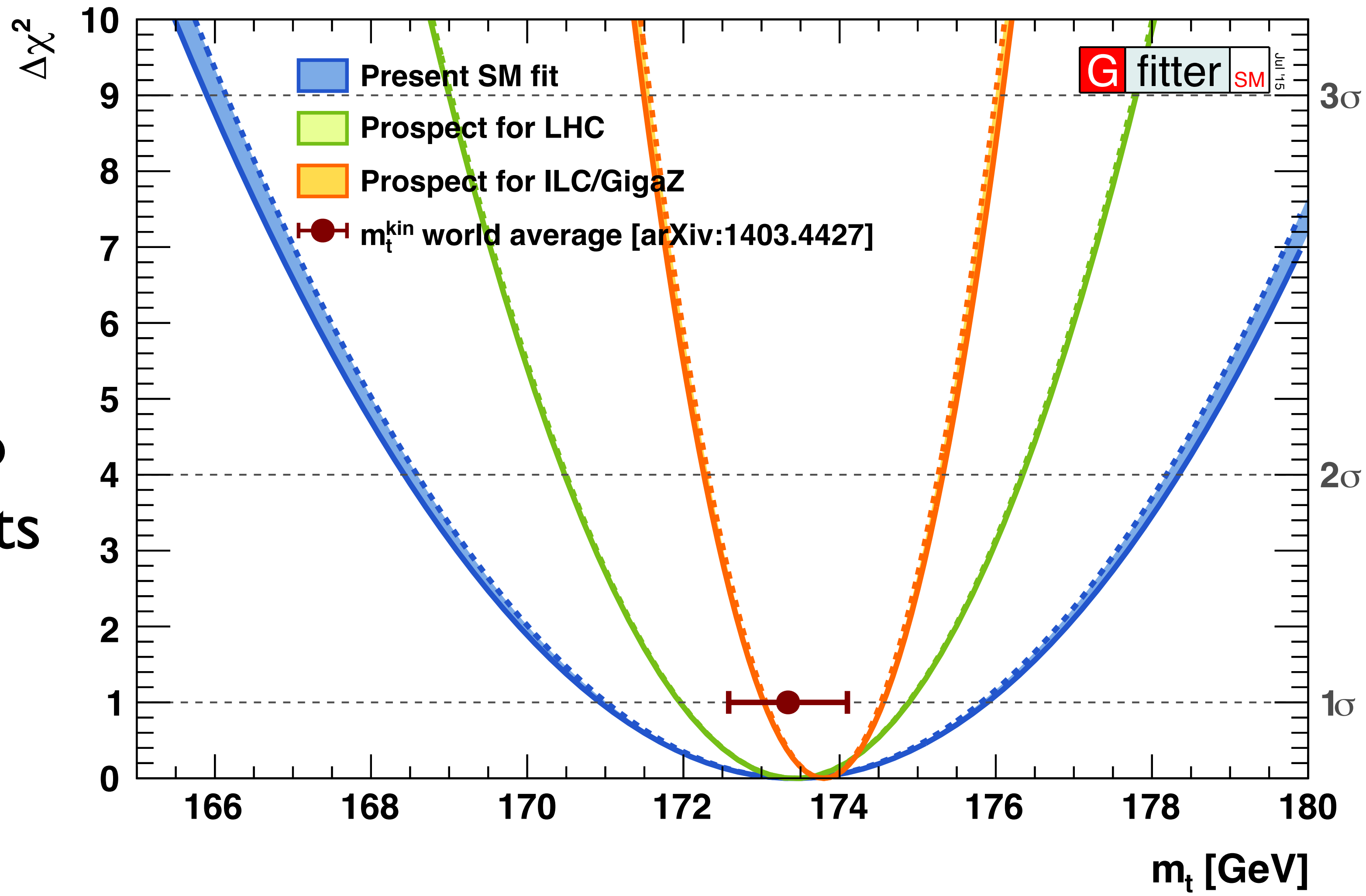


$$m_t = 177.0 \pm 2.3_{M_W,\, \sin^2\theta^f_{\rm eff}} \pm 0.6_{\alpha_s} \pm 0.5_{\Delta\alpha_{\rm had}} \pm 0.4_{M_Z}\ \text{GeV}$$

$$= 177.0 \pm 2.4_{\rm exp} \pm 0.5_{\rm theo}\ \text{GeV}$$

- similar precision as determination from $\sigma_{t\bar{t}}$, good agreement
- dominated by experimental precision

Future: Top Quark Mass



LHC-300 Scenario

- improvement due to improved precision on M_W

ILC Scenario

- Comparable precision due to M_W and $\sin^2\theta^l_{eff}$ measurements (M_W: δm_t = 1 GeV
  $\sin^2\theta^l_{eff}$: δm_t = 0.9 GeV)

Fit Results:

$$\delta m_t = 0.6_{M_W} \oplus 0.5_{M_Z} \oplus 0.3_{\sin^2\theta^f_{\rm eff}} \oplus 0.4_{\Delta\alpha_{\rm had}} \oplus 0.2_{\alpha_s}\ \text{GeV}$$

$$\delta m_t = 0.2_{\rm theo} \oplus 0.7_{\rm exp}\ \text{GeV} = 0.8_{\rm tot}\ \text{GeV}$$

- similar precision as present world average of m_t^{kin} from hadron colliders
- still dominated by experimental precision

Summary of Indirect Predictions

Parameter	Experimental input [$\pm 1\sigma_{\rm exp}$] Present	LHC	ILC/GigaZ	Indirect determination [$\pm 1\sigma_{\rm exp}, \pm 1\sigma_{\rm theo}$] Present	LHC	ILC/GigaZ
M_H [GeV]	0.2	< 0.1	< 0.1	$^{+31}_{-26}, ^{+10}_{-8}$	$^{+20}_{-18}, ^{+3.9}_{-3.2}$	$^{+6.8}_{-6.5}, ^{+2.5}_{-2.4}$
M_W [MeV]	15	8	5	6.0, 5.0	5.2, 1.8	1.9, 1.3
M_Z [MeV]	2.1	2.1	2.1	11, 4	7.0, 1.4	2.5, 1.0
m_t [GeV]	0.8	0.6	0.1	2.4, 0.6	1.5, 0.2	0.7, 0.2
$\sin^2\theta^{\ell}_{\rm eff}$ [10^{-5}]	16	16	1.3	4.5, 4.9	2.8, 1.1	2.0, 1.0
$\Delta\alpha^5_{\rm had}(M_Z^2)$ [10^{-5}]	10	4.7	4.7	42, 13	36, 6	5.6, 3.0
R^0_l [10^{-3}]	25	25	4	–	–	–
$\alpha_S(M_Z^2)$ [10^{-4}]	–	–	–	40, 10	39, 7	6.4, 6.9
$S\|_{U=0}$	–	–	–	0.094, 0.027	0.086, 0.006	0.017, 0.006
$T\|_{U=0}$	–	–	–	0.083, 0.023	0.064, 0.005	0.022, 0.005
κ_V ($\lambda = 3\,$TeV)	0.05	0.03	0.01	0.02	0.02	0.01

Summary of Indirect Predictions

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M_Z [MeV]	2.1	2.1	2.1	11, 4	7.0, 1.4	2.5, 1.0
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$\alpha_s(M_Z^2)$ $[10^{-4}]$	–	–	–	40, 10	39, 7	6.4, 6.9
$S\vert_{U=0}$	–	–	–	0.094, 0.027	0.086, 0.006	0.017, 0.006
$T\vert_{U=0}$	–	–	–	0.083, 0.023	0.064, 0.005	0.022, 0.005
κ_V $(\lambda = 3\,\text{TeV})$	0.05	0.03	0.01	0.02	0.02	0.01

- Theory uncertainty needs to be reduced if we want to achieve the ultimate precision with the LHC!
- Future e^+e^- collider: fantastic possibilities for consistency tests of the SM on loop level and NP constraints

Summary

Uncertainties on M_W

Today

δ_{meas} = 15 MeV

δ_{fit} = 8 MeV

δ_{fit}^{theo} = 5 MeV

LHC-300

δ_{meas} = 8 MeV

δ_{fit} = 6 MeV

δ_{fit}^{theo} = 2 MeV

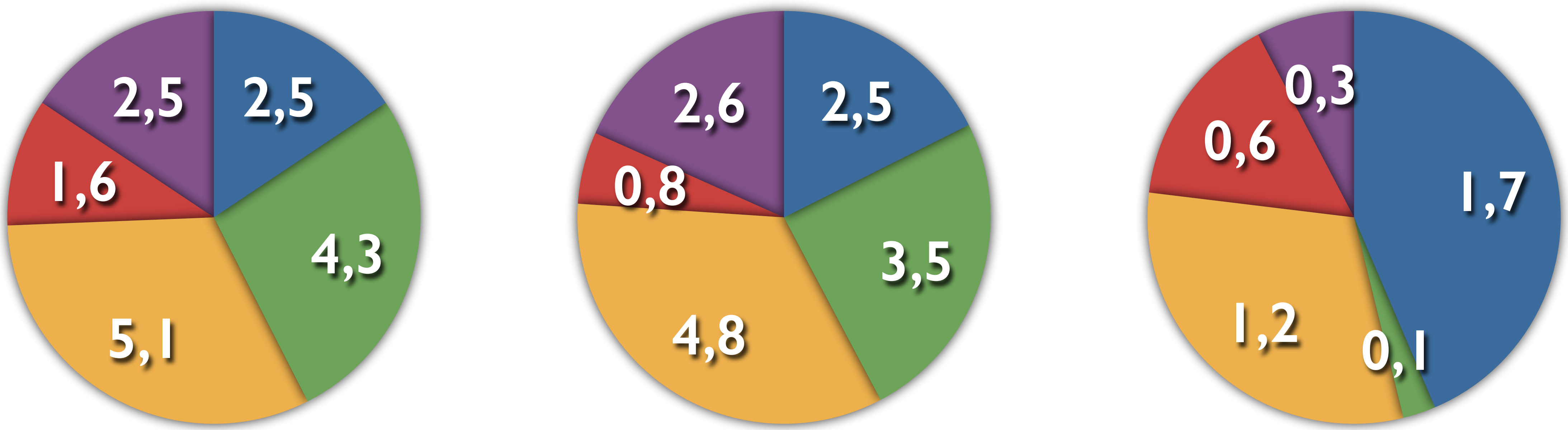
ILC/GigaZ

δ_{meas} = 5 MeV

δ_{fit} = 2 MeV

δ_{fit}^{theo} = 1 MeV

● δM_Z ● δm_{top} ● $\delta \sin^2(\theta^l_{eff})$ ● $\delta\Delta\alpha_{had}$ ● $\delta\alpha_s$



Impact of individual uncertainties on δM_W in fit (numbers in MeV)

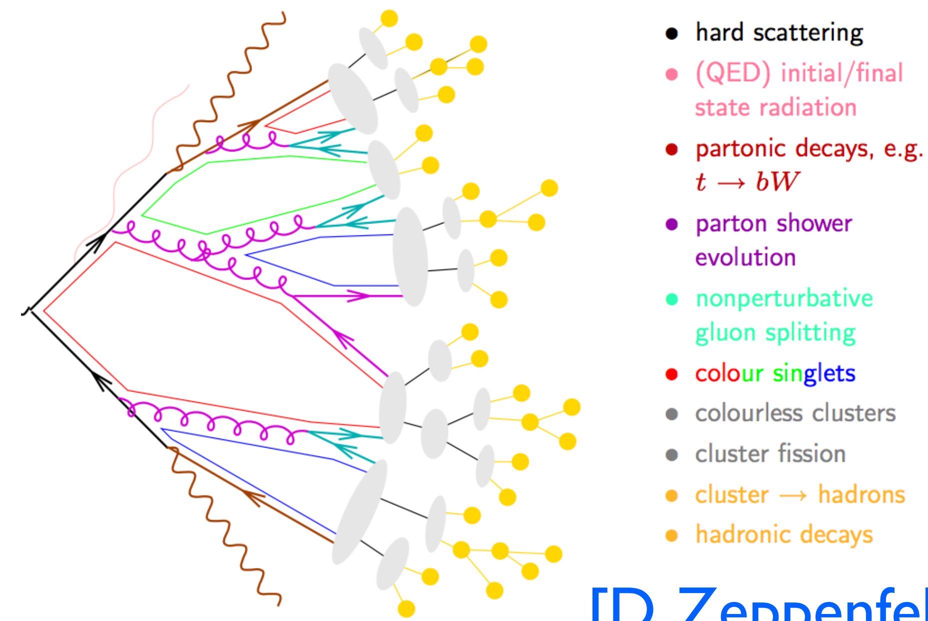
Improved theoretical precision needed already for the LHC-300!

Additional Material

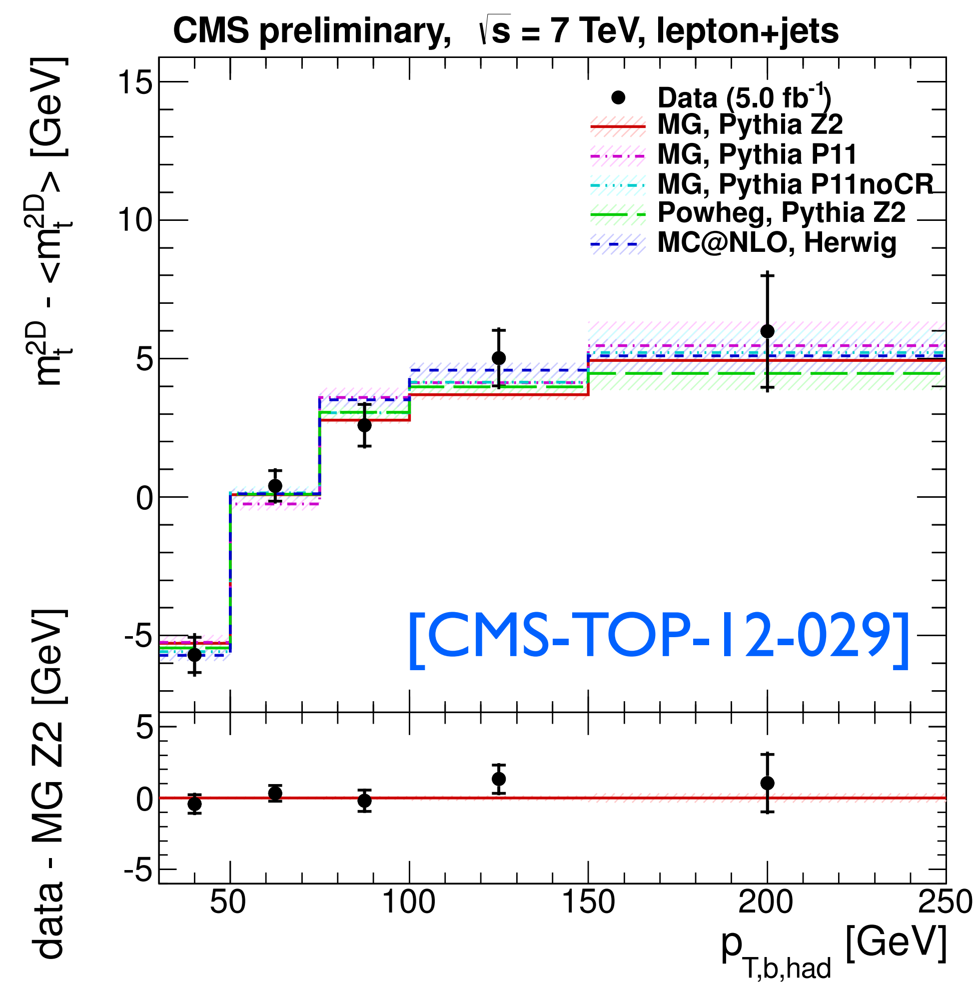
Interpreteation of m_t Measurements

What about accuracy?

- top mass definition
  - EFT, factorization: hard function, universal jet-function, non-pert. soft function [Moch et al, arXiv:1405.4781]
  - MC mass is (most likely) related to the low scale short-distance mass in the jet function [Hoang, arXiv:1412.3649]
  - but: no quantitative statement available
  - relating m_t^{kin} to m_t^{pole} : $\Delta m_t \geq \Lambda_{QCD}$
- colour structure and hadronisation
  - partly included in experimental uncertainties
  - study on kinematic dependencies of m_t
- calculating $m_t(m_t)$ from m_t^{pole}
  - QCD (three-loop): $\Delta m_t \approx 0.02$ GeV
  - EW (two-loop): $\Delta m_t \approx 0.1$ GeV [Kniehl et al., arXiv:1401.1844]



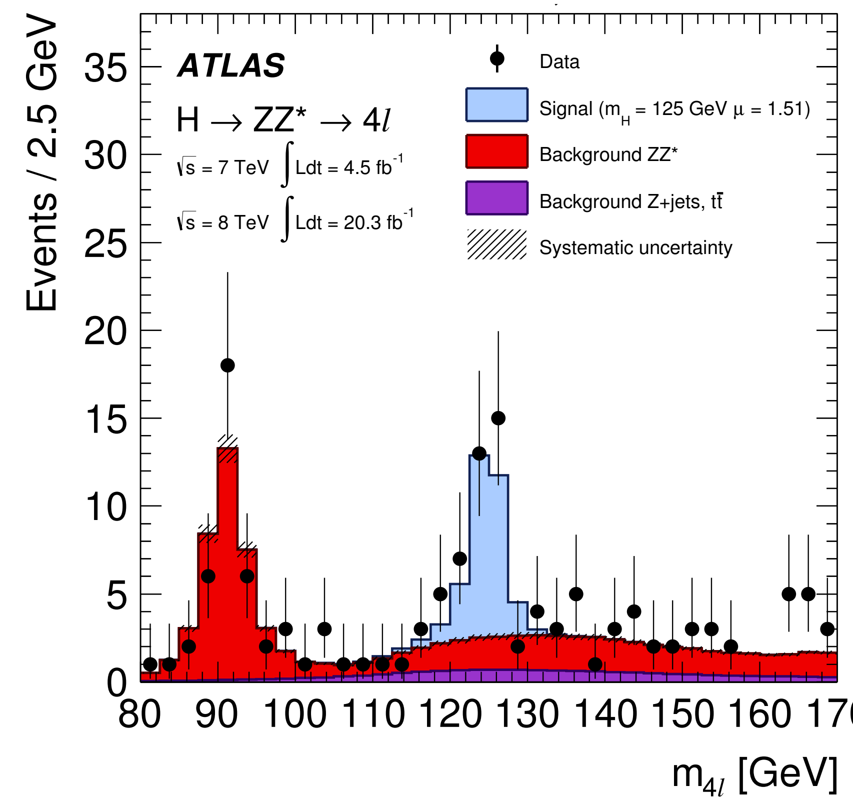
[D. Zeppenfeld]



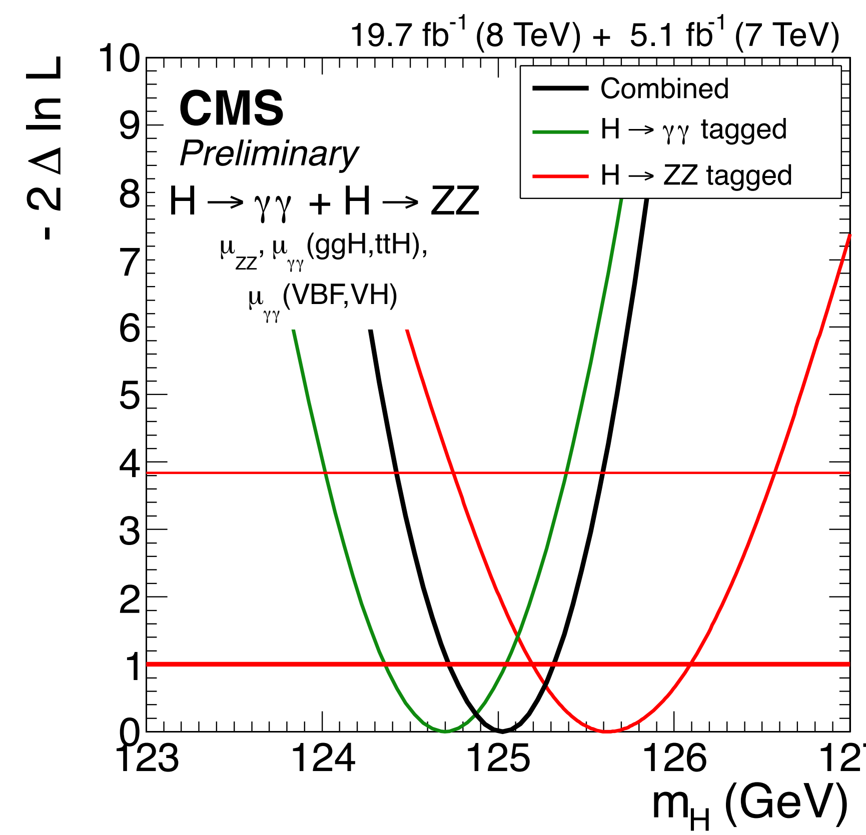
Measurements of M_H

Discovery of a Higgs boson

- cross section times branching ratios, spin, parity: compatible with SM Higgs boson
  - assume it's the SM Higgs boson
    - (or a BSM Higgs boson h in the decoupling region)
  - test the consistency of the SM including it
- best mass measurements: $H\rightarrow\gamma\gamma$, $H\rightarrow 4l$
  - ATLAS: 125.4 ± 0.4 GeV [ATLAS, 1406.3827]
  - CMS: 125.0 ± 0.3 GeV [CMS-PAS-HIG-14-009]
  - weighted average: 125.14 ± 0.24 GeV
    - change between fully uncorrelated and fully correlated systematic uncertainties is minor: δM_H : 0.24 → 0.32 GeV
  - accuracy: 0.2% !
    - sufficient for electroweak fit (more later)



[ATLAS, arXiv:1408.5191]



[CMS-PAS-HIG-14-009]

Measurements of M_W

M_W : key parameter in the SM

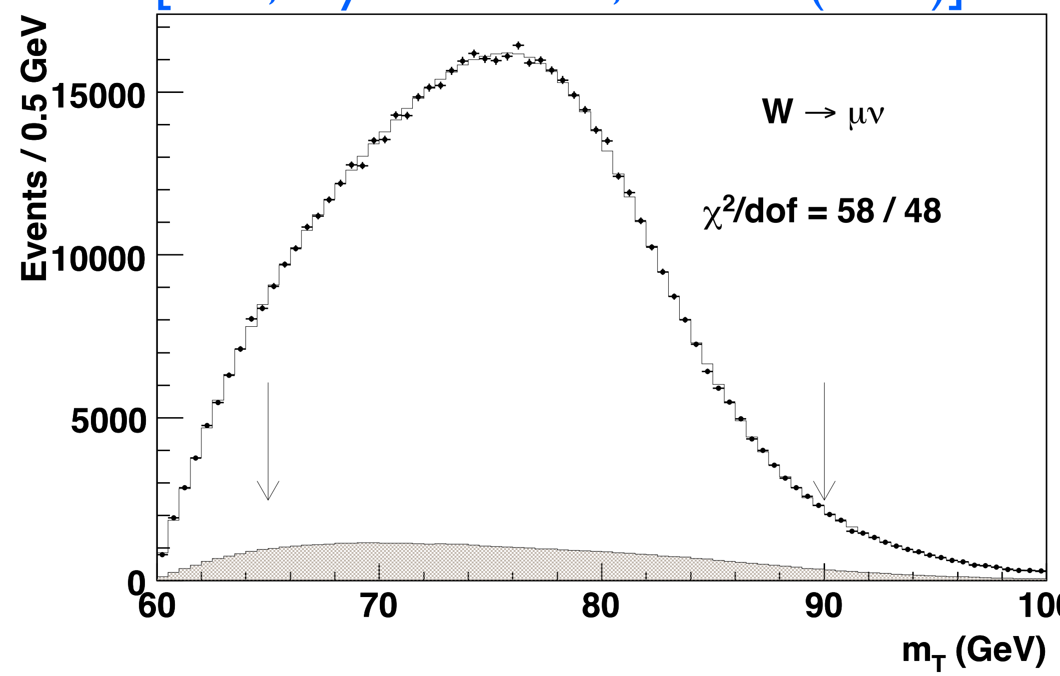
$$\Delta r = -\frac{3\alpha c_W^2}{16\pi s_W^4}\frac{m_t^2}{M_W^2} + \frac{11\alpha}{48\pi s_W^2}\ln\frac{M_H^2}{M_W^2} + \ldots$$

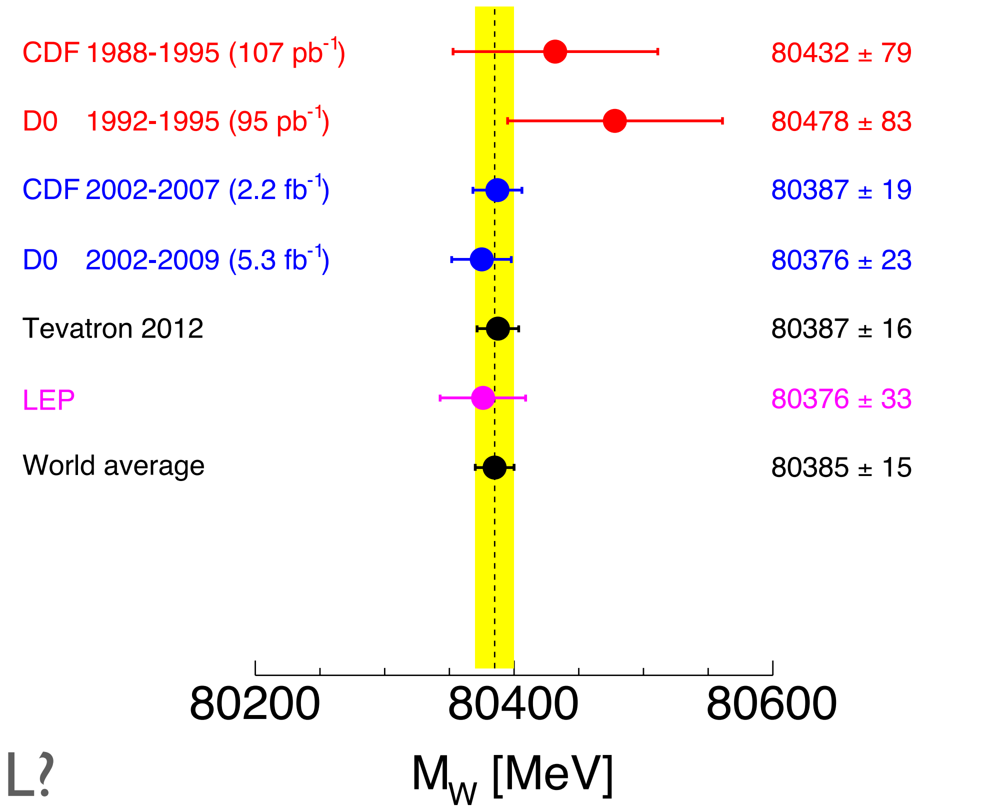
- final LEP-2 measurement (2013):
  - ΔM_W = 33 MeV [ADLO, Phys. Rept. 532:119,2013]
- Tevatron : most precise result so far
  - Jacobean peak in M_T and $p_{T,l}$ in $W \to l\nu$
  - ΔM = 16 MeV, accuracy: 0.02% !!
  - crucial: lepton energy and resolution, PDFs
- LHC : no result so far
  - (optimistic) scenarios: [arXiv:1310.6708]

ΔM_W [MeV]	LHC		
$\sqrt{s}$ [TeV]	8	14	14
$\mathcal{L}$[fb^{-1}]	20	300	3000
Total	15	8	5

  - very challenging
    - PDFs, momentum scale, hadronic recoil, pile-up at high L?

[CDF, Phys. Rev. D 89, 072003 (2014)]





[CDF, D0, Phys. Rev. D 88, 052018 (2013)]

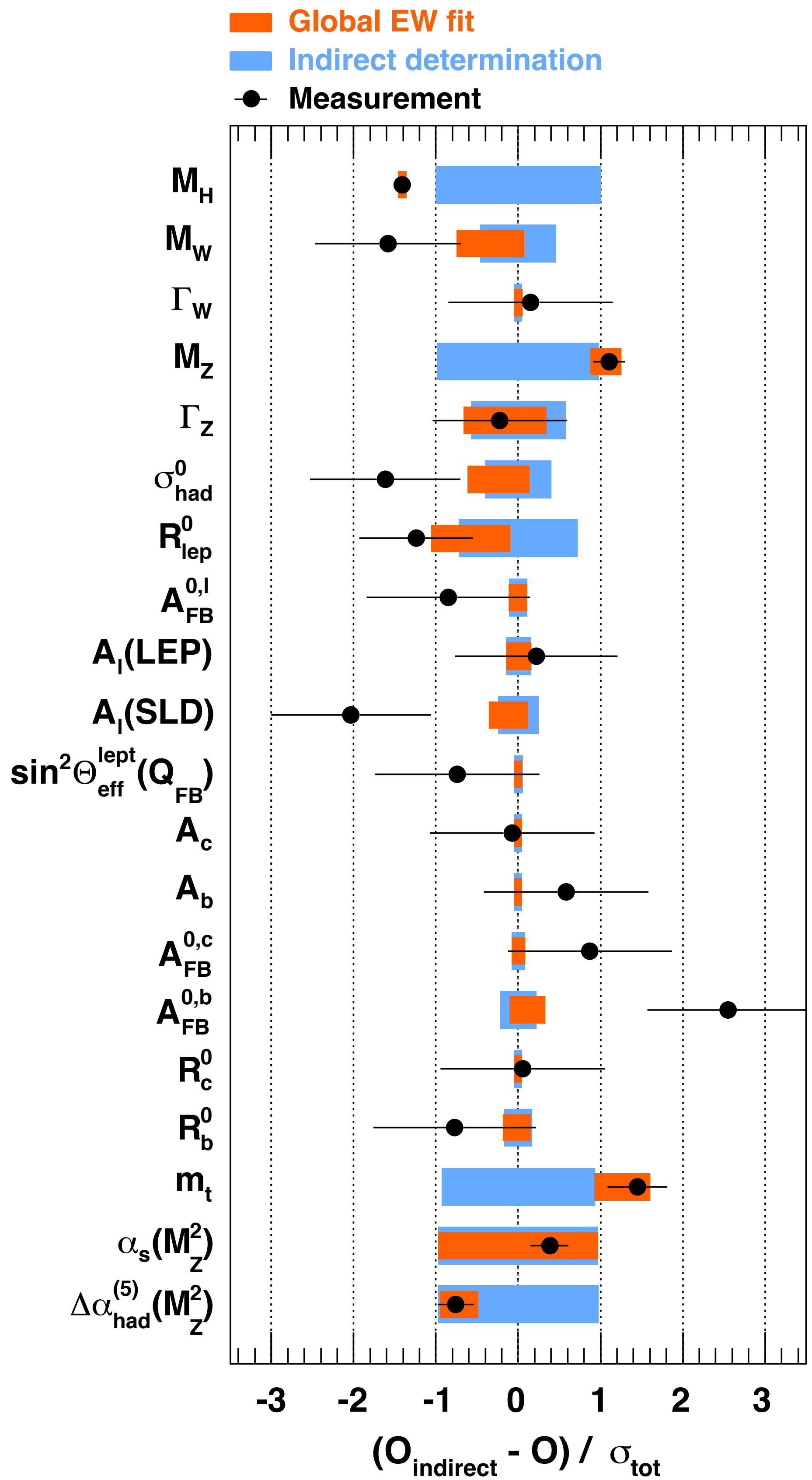
SM Fit Results

black: direct measurement (data)

orange: full fit

light-blue: fit excluding input from row

- goodness of fit, p-value:
  χ^2_{min} = 17.8 Prob(χ^2_{min}, 14) = 21%
  Pseudo experiments: 21 ± 2 (theo)%
  - χ^2_{min}(Z widths in 1-loop) = 18.0
  - χ^2_{min}(no theory uncertainties) = 18.2
- no individual value exceeds 3σ
- largest deviations in b-sector:
  - $A^{0,b}_{FB}$ with 2.5σ
    → largest contribution to χ^2
- small pulls for M_H, M_Z
  - input accuracies exceed fit requirements

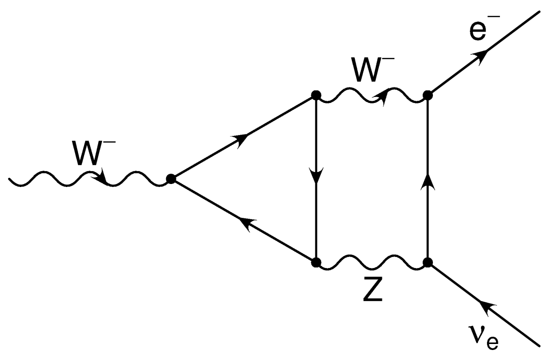


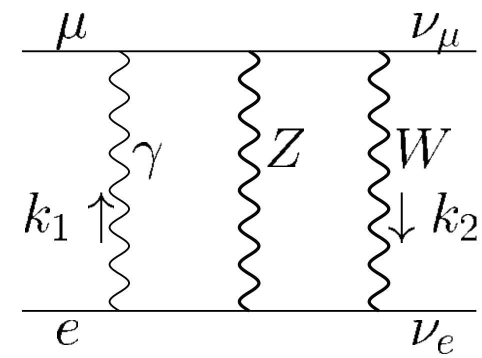
Calculation of M_W

- Full EW one- and two-loop calculation of fermionic and bosonic contributions
- One- and two-loop QCD corrections and leading terms of higher order corrections
- Results for Δr include terms of order $O(\alpha)$, $O(\alpha\alpha_s)$, $O(\alpha\alpha_s^2)$, $O(\alpha^2_{\mathrm{ferm}})$, $O(\alpha^2_{\mathrm{bos}})$, $O(\alpha^2\alpha_s m_t^4)$, $O(\alpha^3 m_t^6)$
- Uncertainty estimate:
  - missing terms of order $O(\alpha^2\alpha_s)$: about 3 MeV (from $O(\alpha^2\alpha_s m_t^4)$)
  - electroweak three-loop correction $O(\alpha^3)$: < 2 MeV
  - three-loop QCD corrections $O(\alpha\alpha_s^3)$: < 2 MeV
  - Total: $\delta M_W \approx 4$ MeV

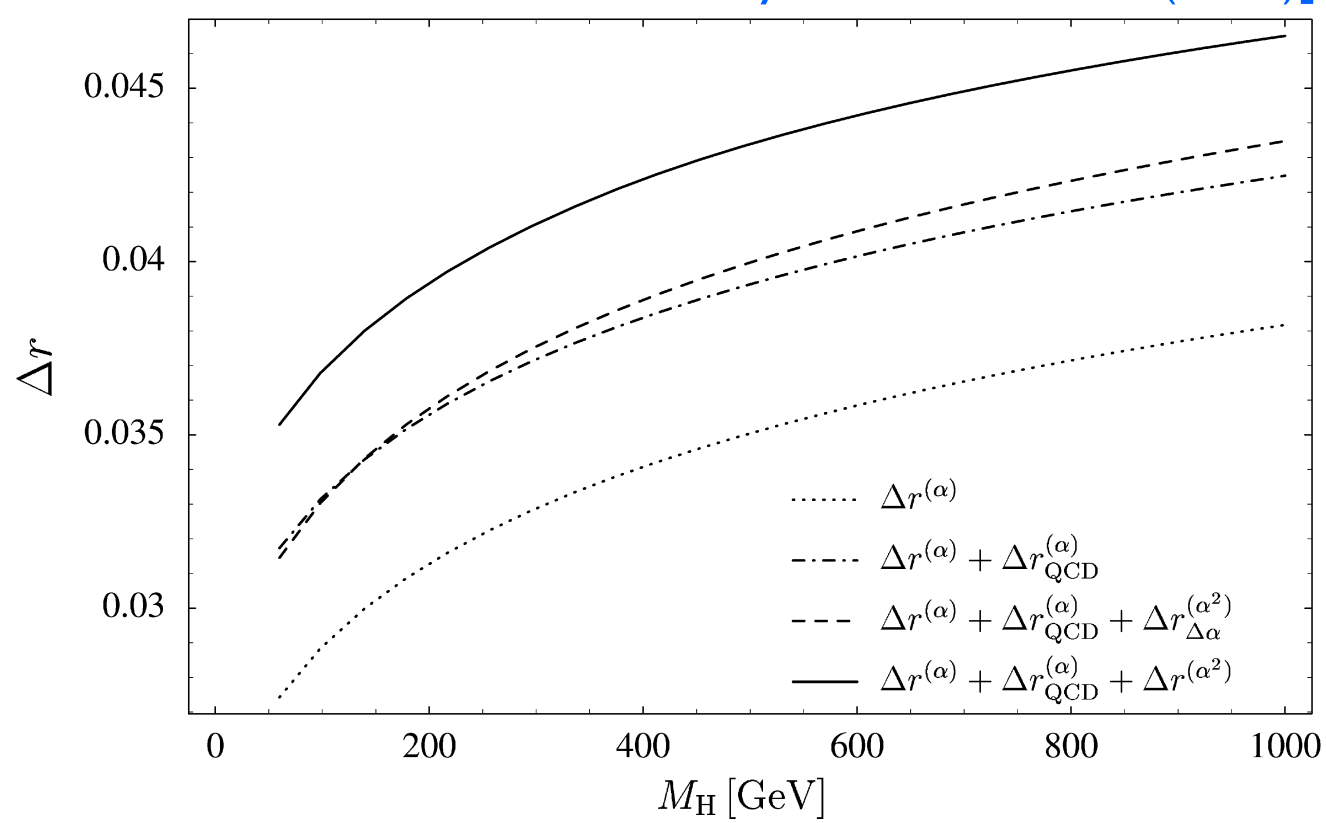
[M Awramik et al., Phys. Rev. D69, 053006 (2004)]

[M Awramik et al., Phys. Rev. Lett. 89, 241801 (2002)]





A Freitas et al., Phys. Lett. B495, 338 (2000)]



Calculation of $\sin^2(\theta^l_{eff})$

- Effective mixing angle:

$$\sin^2\theta^{\rm lept}_{\rm eff} = \left(1 - M^2_{\rm W}/M^2_{\rm Z}\right)(1+\Delta\kappa)$$

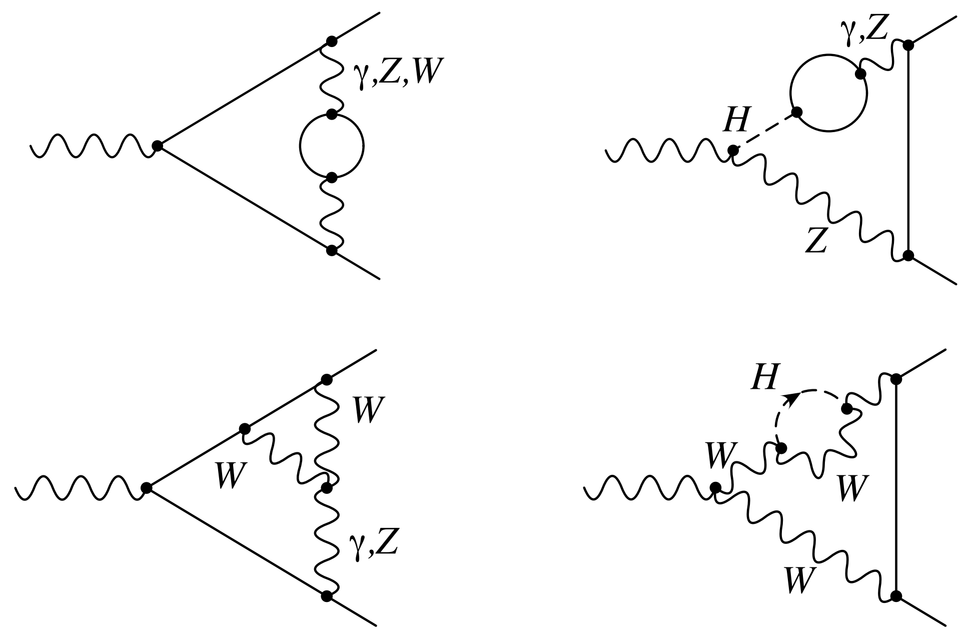
- Two-loop EW and QCD correction to $\Delta\kappa$ known, leading terms of higher order QCD corrections
- fermionic two-loop correction about 10^{-3}, whereas bosonic one 10^{-5}
- Uncertainty estimate obtained with different methods, geometric progression:

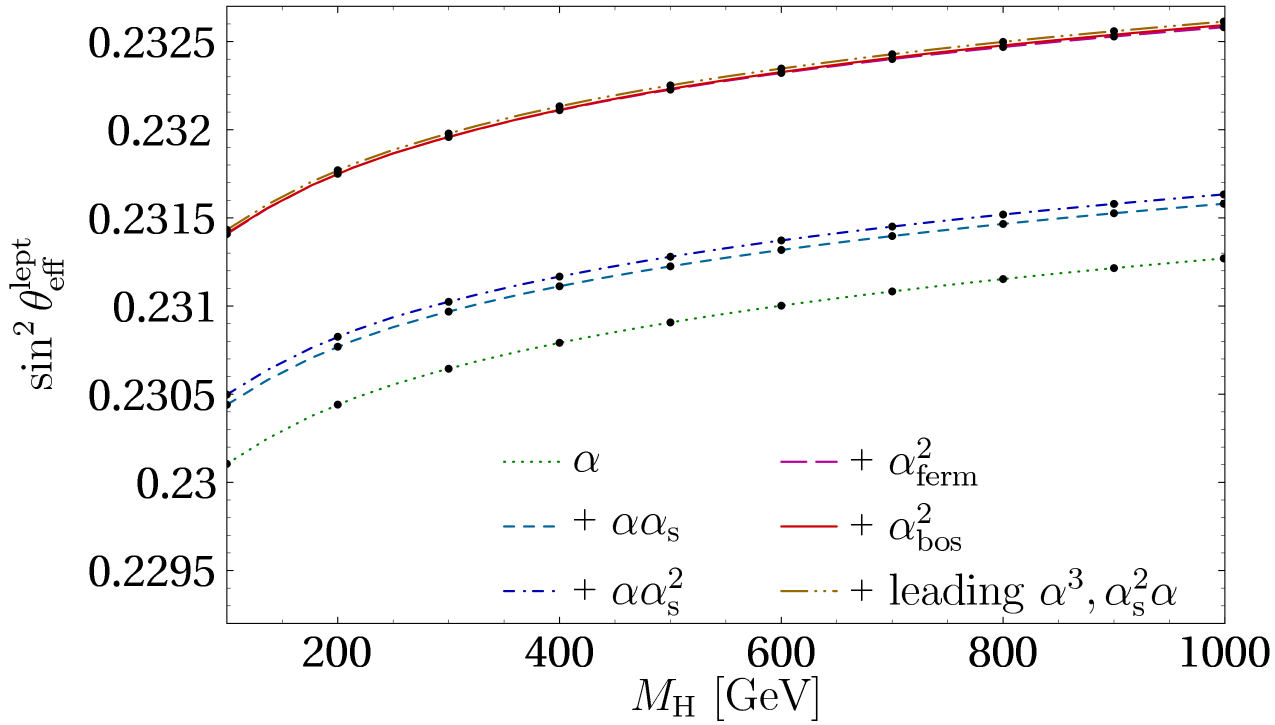
$$\mathcal{O}(\alpha^2\alpha_{\rm s}) = \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}\,\mathcal{O}(\alpha\alpha_{\rm s}).$$

$\mathcal{O}(\alpha^2\alpha_{\rm s})$ beyond leading $m_{\rm t}^4$	$3.3\ldots2.8\times10^{-5}$
$\mathcal{O}(\alpha\alpha_{\rm s}^3)$	$1.5\ldots1.4$
$\mathcal{O}(\alpha^3)$ beyond leading $m_{\rm t}^6$	$2.5\ldots3.5$

Total: $\delta\sin^2\theta^l_{eff} \approx 4.7\ 10^{-5}$

[M Awramik et al, Phys. Rev. Lett. 93, 201805 (2004)]
[M Awramik et al., JHEP 11, 048 (2006)]



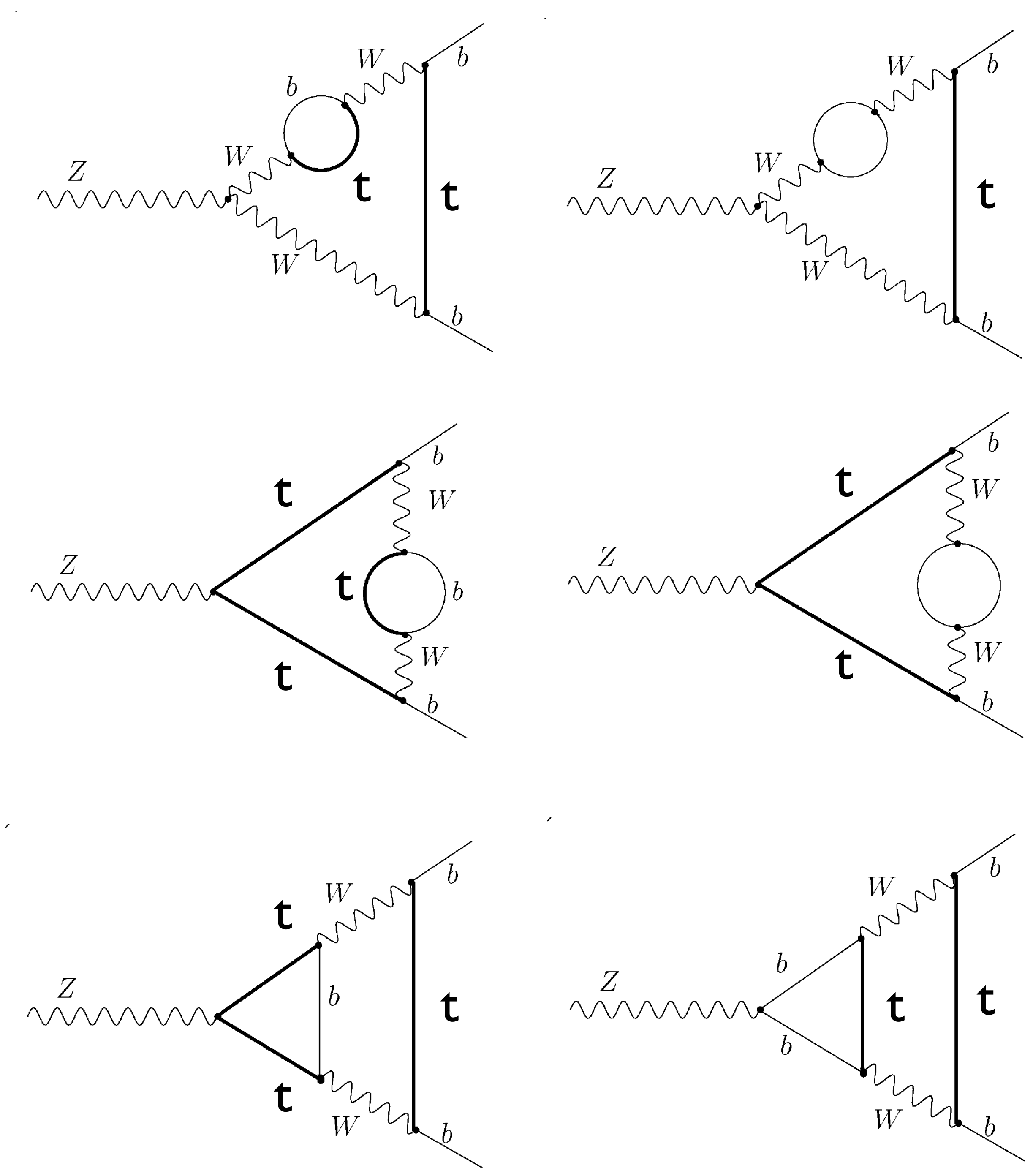


Calculation of $\sin^2(\theta^{bb}_{eff})$

[M Awramik et al, Nucl. Phys. B813, 174 (2009)]

- Calculation of $\sin^2\theta_{eff}$ for b-quarks more involved, because of top quark propagators in the $Z \rightarrow b\bar{b}$ vertex
- Investigation of known discrepancy between $\sin^2\theta_{eff}$ from leptonic and hadronic asymmetry measurements
- Two-loop EW correction only recently completed, effect of $O(10^{-4})$
- Now $\sin^2\theta^{bb}_{eff}$ known at the same order as $\sin^2\theta_{eff}$ for leptons and light quarks
- Uncertainty assumed to be of same size as for $\sin^2\theta_{eff}$:

$$\delta\sin^2\theta^{bb}_{eff} \approx 4.7\ 10^{-5}$$



Calculation of R^0_b

Full two-loop calculation of $Z\to b\bar{b}$ [A. Freitas et al., JHEP 1208, 050 (2012) Erratum ibid. 1305 (2013) 074]

- The branching ratio $R^0{}_b$: partial decay width of $Z\to b\bar{b}$ and $Z\to q\bar{q}$

$$R_b \equiv \frac{\Gamma_b}{\Gamma_{\rm had}} = \frac{\Gamma_b}{\Gamma_d+\Gamma_u+\Gamma_s+\Gamma_c+\Gamma_b} = \frac{1}{1+2(\Gamma_d+\Gamma_u)/\Gamma_b}$$

- Contribution of same terms as in the calculation of $\sin^2\theta^{bb}{}_{\rm eff}$
  → cross-check the two results, found good agreement

- Two-loop corrections small compared to experimental uncertainty ($6.6\cdot 10^{-4}$)

	1-loop EW and QCD correction to FSR	2-loop EW correction	2-loop EW and 2+3-loop QCD correction to FSR	1+2-loop QCD correction to gauge boson selfenergies
$M_{\rm H}$ [GeV]	$\mathcal{O}(\alpha)+{\rm FSR}_{\alpha,\alpha_s,\alpha_s^2}$ $[10^{-4}]$	$\mathcal{O}(\alpha^2_{\rm ferm})$ $[10^{-4}]$	$\mathcal{O}(\alpha^2_{\rm ferm})+{\rm FSR}_{\alpha_s^3,\alpha\alpha_s,m_b^2\alpha_s,m_b^4}$ $[10^{-4}]$	$\mathcal{O}(\alpha\alpha_s,\alpha\alpha_s^2)$ $[10^{-4}]$
100	−35.66	−0.856	−2.496	−0.407
200	−35.85	−0.851	−2.488	−0.407
400	−36.09	−0.846	−2.479	−0.406

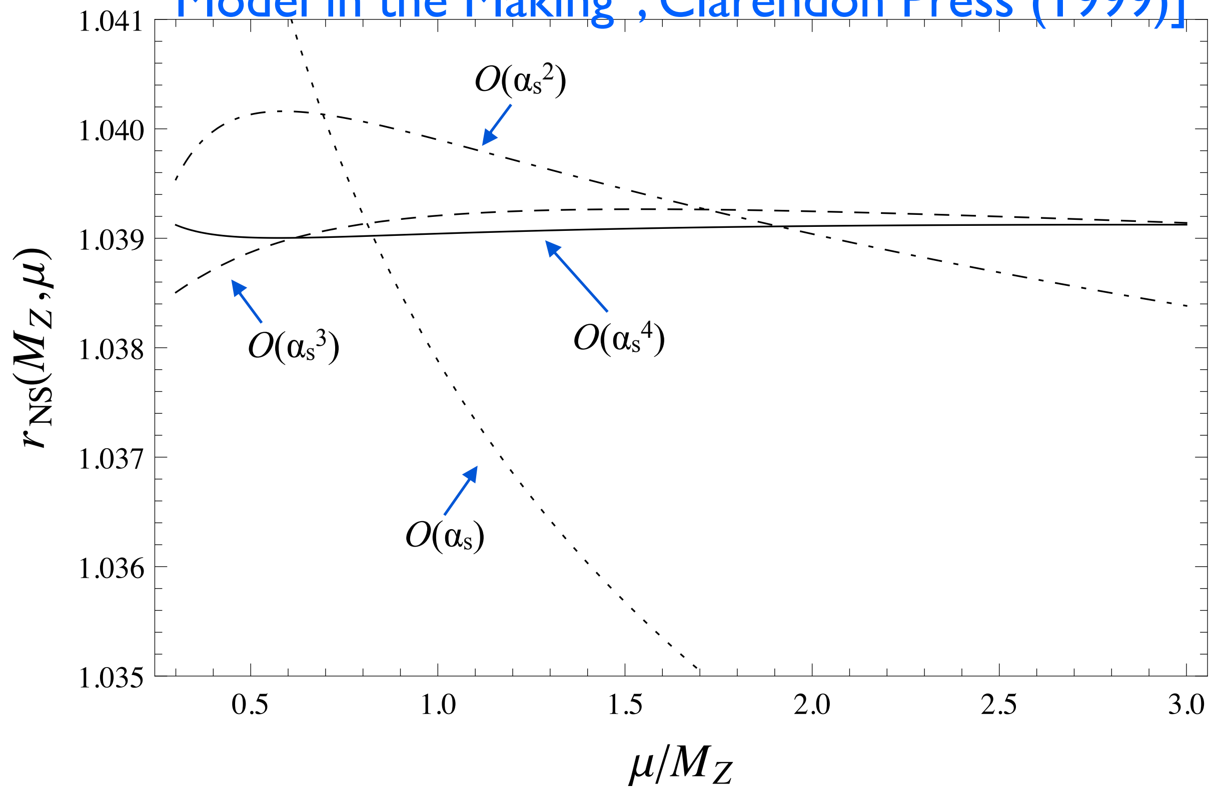
Radiator Functions

- Partial widths are defined inclusively: they contain QCD and QED contributions
- Corrections can be expressed as radiator functions $R_{A,f}$ and $R_{V,f}$

$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left(|g_{A,f}|^2 R_{A,f} + |g_{V,f}|^2 R_{V,f}\right)^2$$

- High sensitivity to the strong coupling α_s
- Full four-loop calculation of QCD Adler function available (N^3LO)
- Much reduced scale dependence
- Theoretical uncertainty of 0.1 MeV, compare to experimental uncertainty of 2.0 MeV

[D. Bardin, G. Passarino, "The Standard Model in the Making", Clarendon Press (1999)]



[P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)]
[P. Baikov et al Phys. Rev. Lett. 104, 132004 (2010)]

Modified Higgs Couplings

Study of potential deviations of Higgs couplings from SM

- BSM modelled as extension of SM through effective Lagrangian
  - Leading corrections only

$$L_V = \frac{h}{v}\left(2\kappa_V m_W^2 W_\mu W^\mu + \kappa_V m_Z^2 Z_\mu Z^\mu\right)$$

$$L_F = -\frac{h}{v}\left(\kappa_F m_t \bar{t}t + \kappa_F m_b \bar{b}b + \kappa_F m_\tau \bar{\tau}\tau\right)$$

- Benchmark model:
  - Scaling of Higgs-vector boson (κ_V) and Higgs-fermion couplings (κ_F)
  - No additional loops in the production or decay of the Higgs, no invisible Higgs decays and undetectable width
- Main effect on EWPO due to modified Higgs coupling to gauge bosons (κ_V)
  - Involving the longitudinal d.o.f.
- Most BSM models: $\kappa_V < 1$
- Additional Higgses typically give positive contribution to M_W