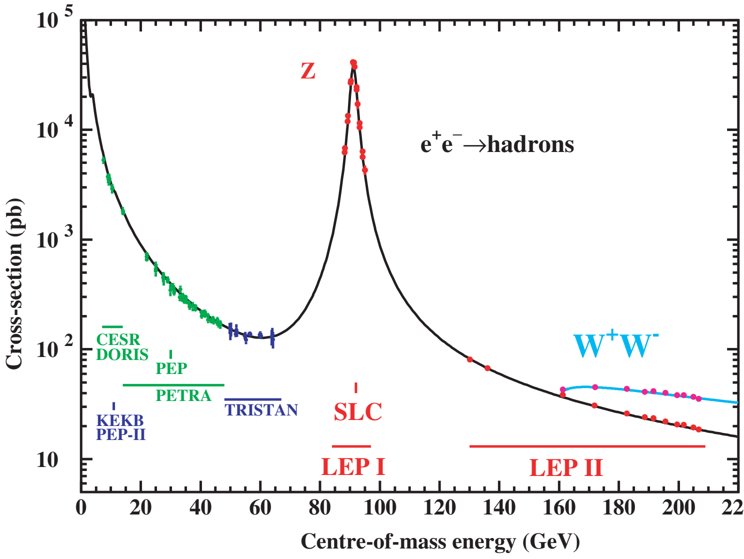
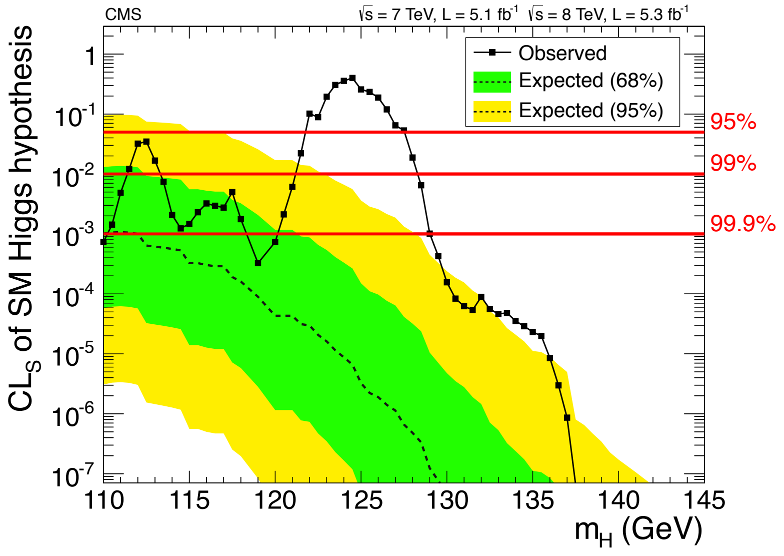
# The Electroweak Fit of the Standard Model with a Higgs Boson at 126 GeV



Roman Kogler
(Universität Hamburg)

for the Gfitter group

LEXI Meeting
Hamburg, Oct 11, 2012

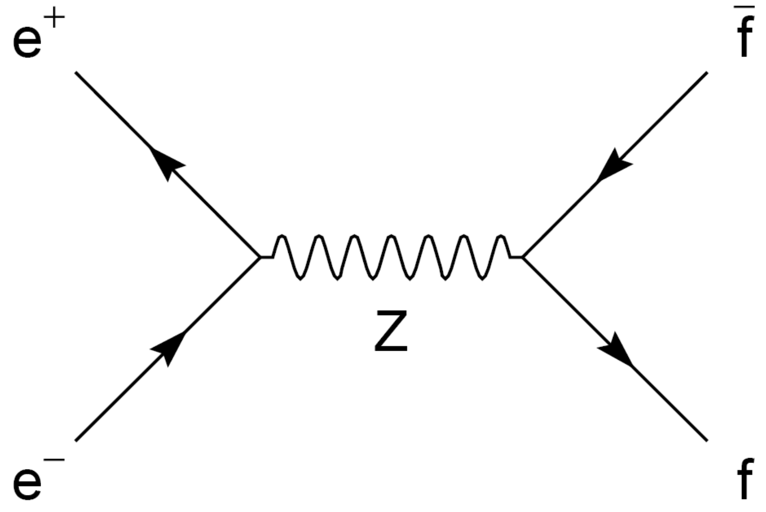


The Gfitter group: M. Baak (CERN), M. Göbel (Univ. Hamburg, DESY), J. Haller (Univ. Hamburg), A. Höcker (CERN), D. Kennedy (Univ. Hamburg, DESY), R. K. (Univ. Hamburg), K. Mönig (DESY), M. Schott (CERN) J. Stelzer (DESY)

# Predictive Power of the SM

## Tree level relations for $Z \rightarrow f \bar{f}$

$$g_{V,f}^{(0)} \equiv g_{L,f}^{(0)} + g_{R,f}^{(0)} = I_3^f - 2Q^f \sin^2\theta_W$$
$$g_{A,f}^{(0)} \equiv g_{L,f}^{(0)} - g_{R,f}^{(0)} = I_3^f \,,$$



with the weak mixing angle:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

Electroweak unification connects the electromagnetic and the weak coupling strengths

$$G_F = \frac{\pi\alpha}{\sqrt{2}(M_W^{(0)})^2\left(1 - \frac{(M_W^{(0)})^2}{M_Z^2}\right)}$$

...and $M_W$ can be expressed in terms of $M_Z$ and $G_F$

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha}{G_F M_Z^2}}\right)$$

Electroweak sector of SM is given by three free parameters, for example α, $G_F$ and $M_Z$

# Radiative Corrections

## Modification of propagators and vertices

- Parametrisation of radiative corrections: electroweak form factors ρ, κ, Δr
- Effective couplings at the Z-pole:

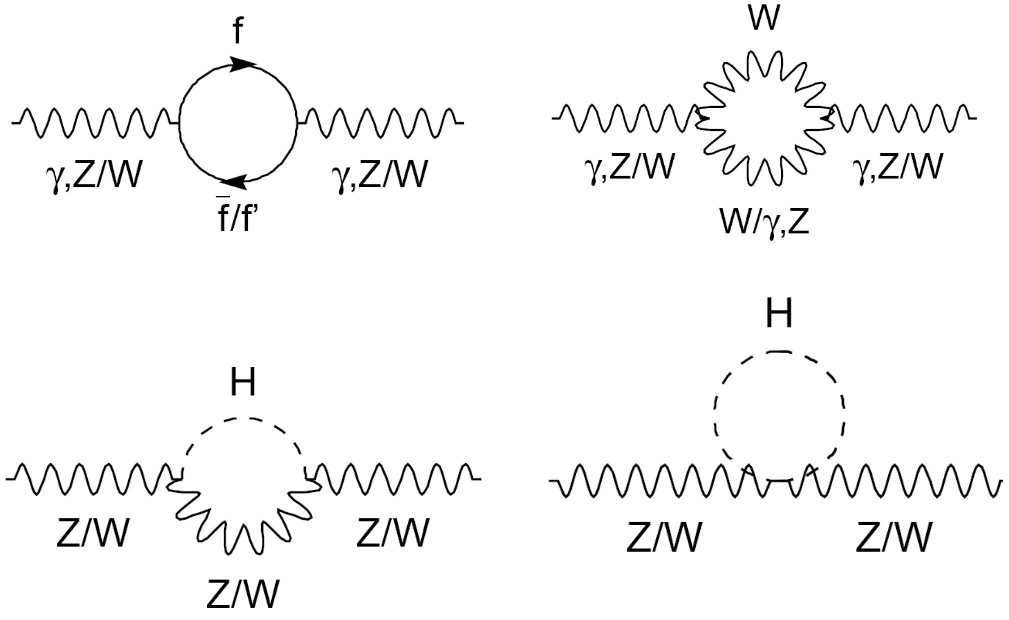
$$g_{V,f} = \sqrt{\rho_Z^f}\left(I_3^f - 2Q^f \sin^2\theta_{\text{eff}}^f\right)$$

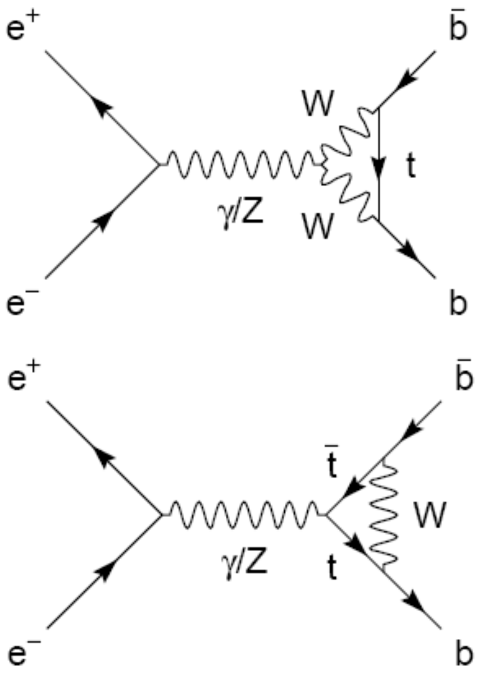
$$g_{A,f} = \sqrt{\rho_Z^f} I_3^f$$

$$\sin^2\theta_{\text{eff}}^f = \kappa_Z^f \sin^2\theta_W$$

- Mass of the W boson:

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha(1-\Delta r)}{G_F M_Z^2}}\right)$$





- ρ, κ, Δr depend nearly quadratically on $m_t$ and logarithmically on $M_H$

Precision tests and constraints of the SM

# Electroweak Fits - History

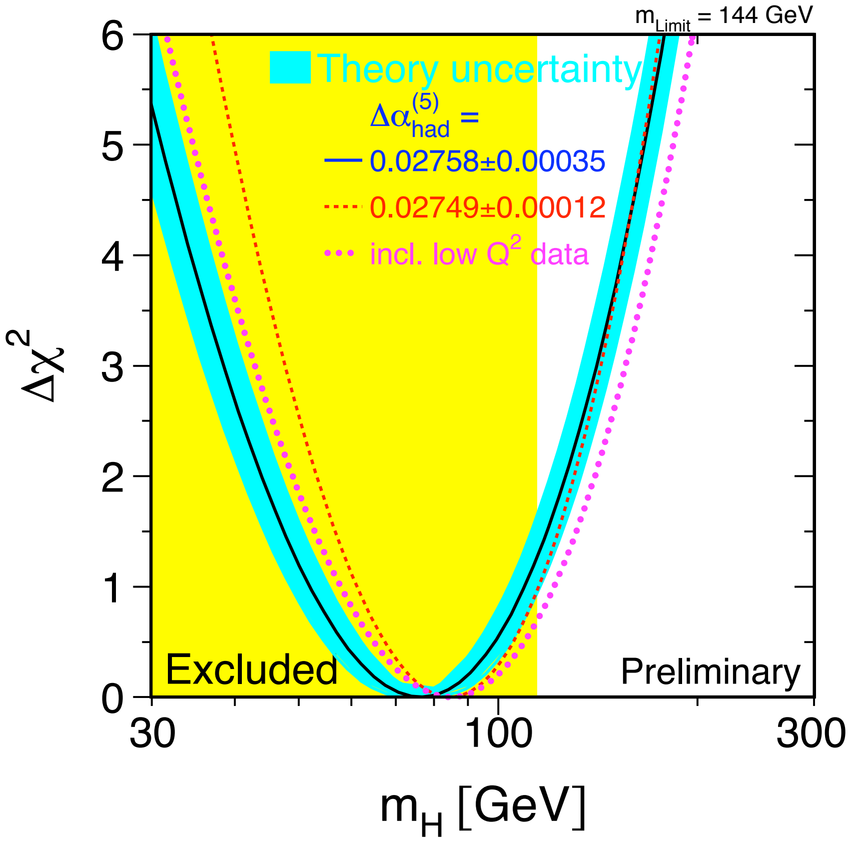
## Electroweak Fits to precision data have a long history

- Huge amount of work to precisely understand loop corrections in the SM
- Precise SM predictions and measurements

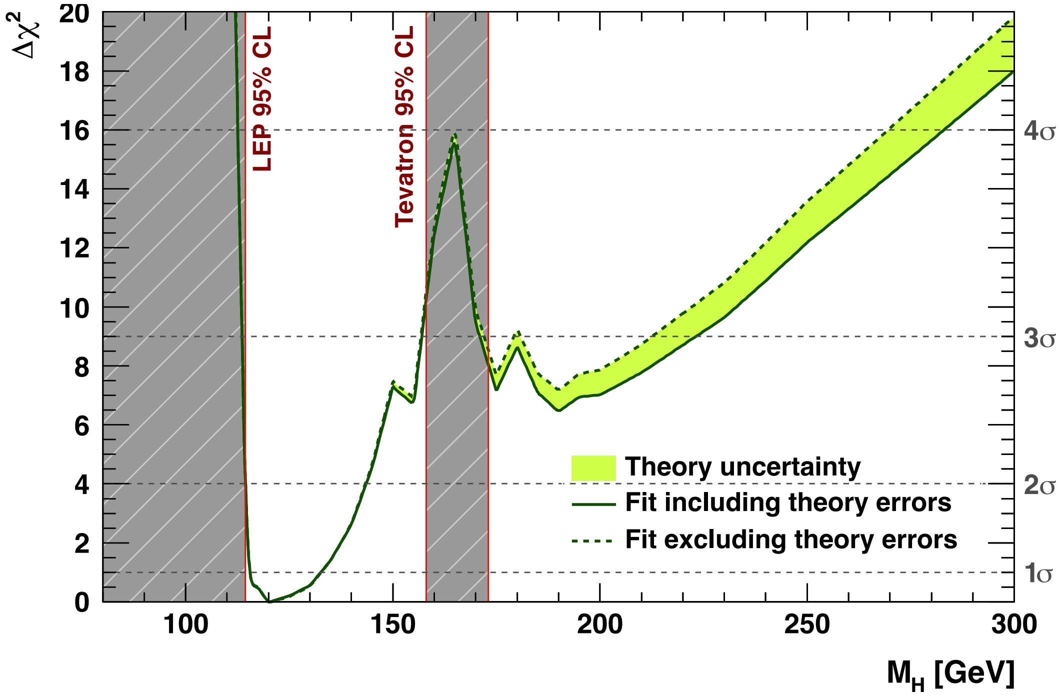
## Electroweak Fits routinely performed by many groups

- D. Bardin et al. (ZFITTER), G. Passarino et al. (TOPAZ0), M. Grünewald et al. (LEP EWWG), J. Erler (GAPP), M. Baak et al. (Gfitter),...
- Many important results obtained, e.g. constraints on the mass of the Higgs boson

The LEP EWWG, arXiv: 0712.9029



Gfitter group, EPJC 72, 2003 (2012)



# The Gfitter Project

G fitter **A Generic Fitter Project for HEP Model Testing**

- Modular framework for involved fitting problems in the LHC era
- Coherent treatment of statistical, systematic and theoretical uncertainties together with possible correlations
- Different packages/plug-ins possible

G fitter SM **A Gfitter package for the global electroweak fit**

- Complete implementation of SM predictions of precision observables
- State of the art calculations used, in particular:
  - Full calculation of the QCD Adler function (massless and massive terms) in N$^3$LO [P.A. Baikov et al., Phys. Rev. Lett. 101 (2008) 012022, Phys. Rev. Lett. 108, 222003 (2012)]
  - Full two-loop correction (NNLO) to $R^0_b$ [A. Freitas et al., JHEP 1208, 050 (2012)]

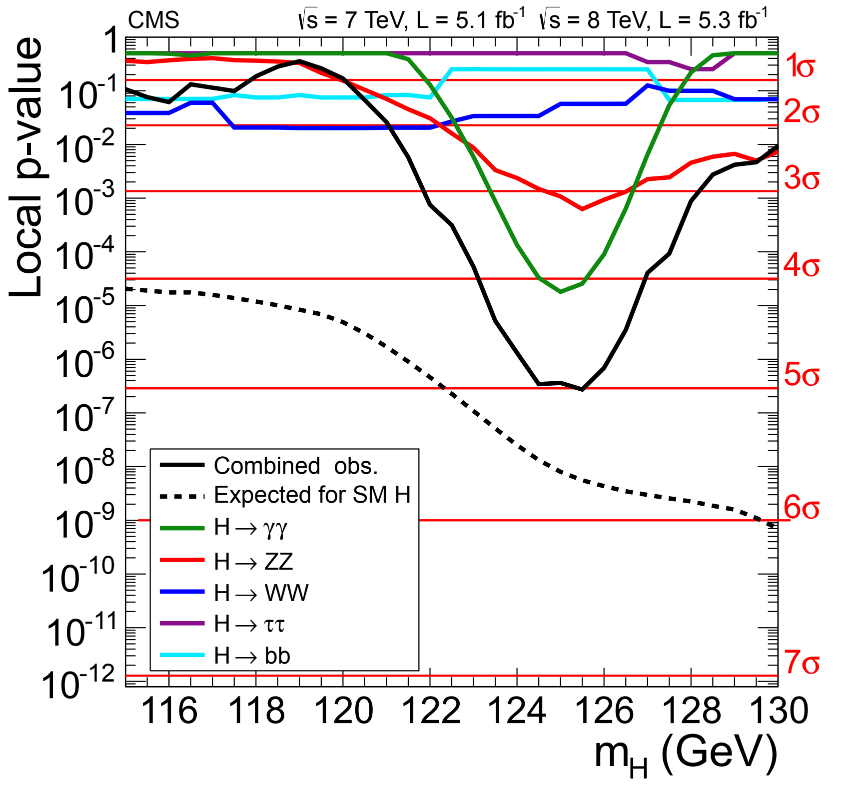
www.cern.ch/gfitter

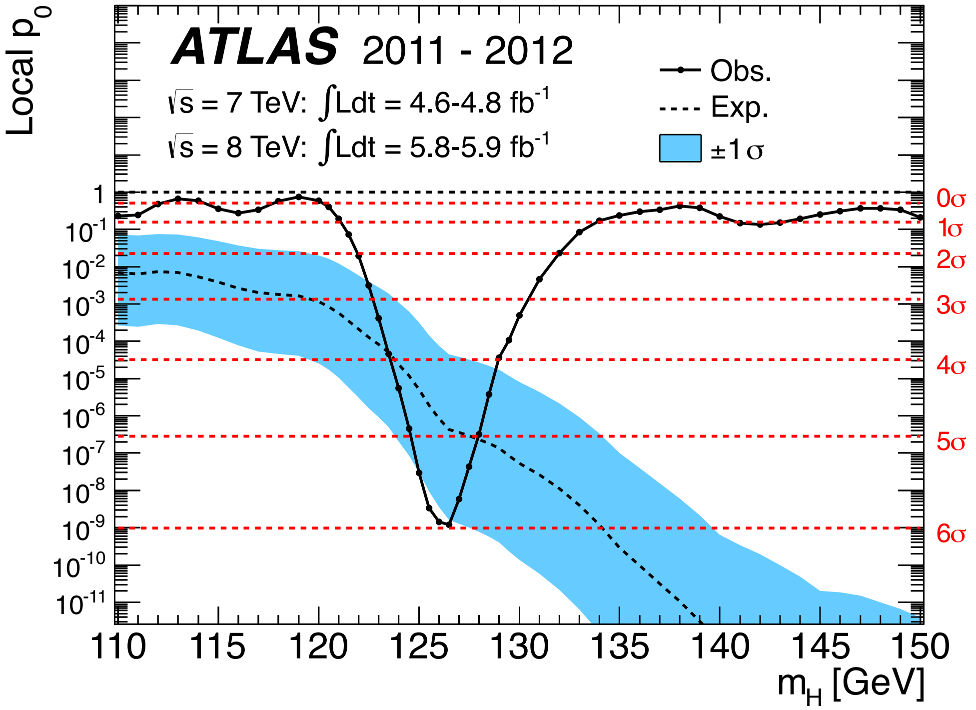
# This Year's Discovery

**ATLAS and CMS have reported the discovery of a new boson**

- The cross section and branching ratios are compatible with the SM Higgs boson
- Measured mass:
  ATLAS: 126.0 ± 0.4 (stat) ± 0.4 (sys) GeV
  CMS: 125.3 ± 0.4 (stat) ± 0.4 (sys) GeV
- Assume that it is the Higgs boson, then $M_H = 125.7 \pm 0.4$ GeV
- Difference between fully uncorrelated and fully correlated systematic uncertainties: uncertainty on $M_H$ 0.4 → 0.5 GeV

The SM is for the first time fully overconstrained → test its consistency





# Experimental Input

## Observables:

- Z-pole observables: LEP/SLD results
  [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
- $M_W$ and $\Gamma_W$: LEP/Tevatron [arXiv:1204:0042]
- $m_t$ : Tevatron [arXiv:1207:1069]
- $\Delta\alpha_{had}^{(5)}(M_Z)$ [M. Davier et al., EPJC 71, 1515 (2011)]
- $\overline{m}_c$, $\overline{m}_b$: world averages
  [PDG, J. Phys. G33, 1 (2006)]
- $M_H$: LHC [arXiv:1207.7214 , arXiv:1207.7235]

## Free fit parameters:

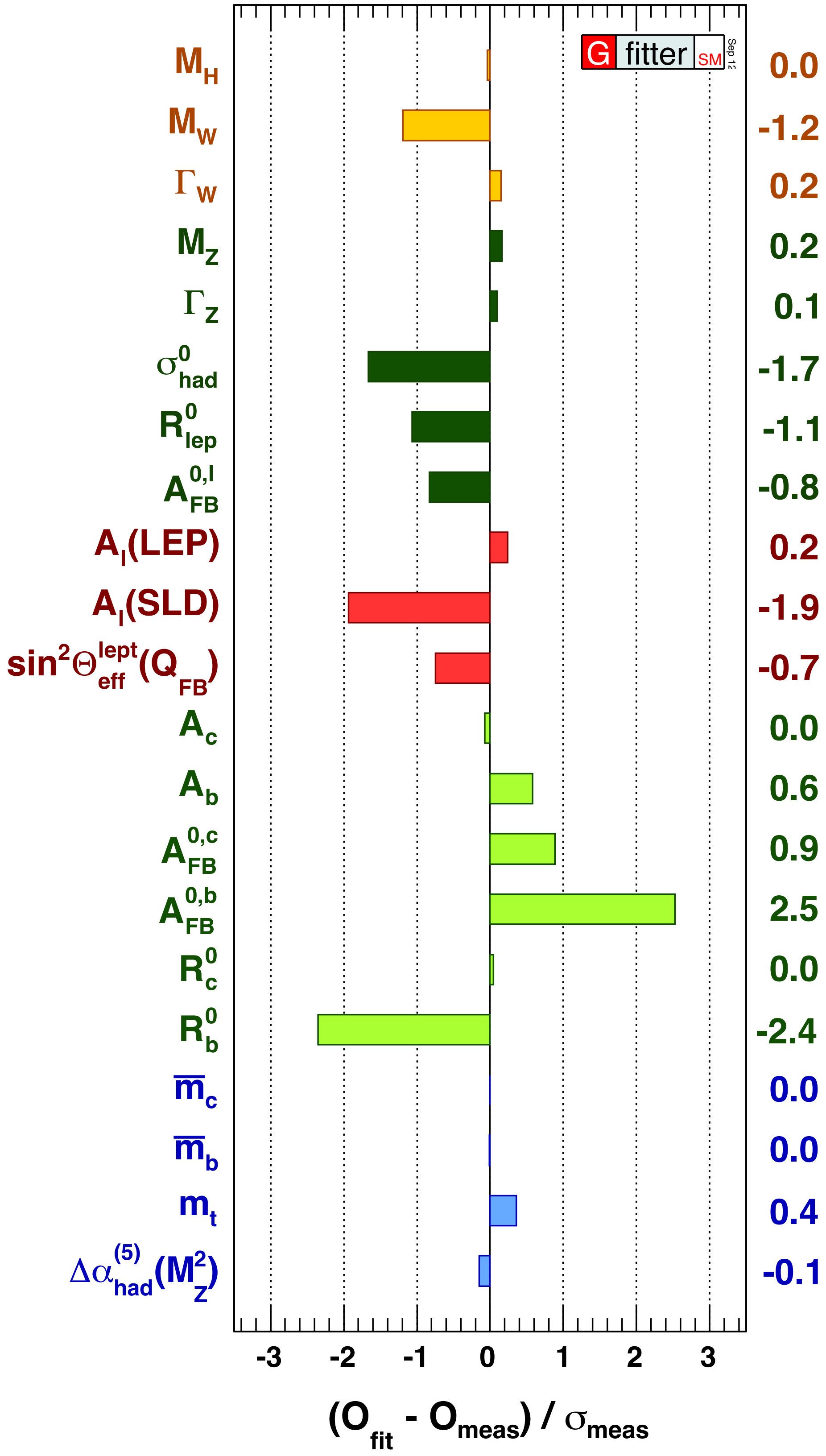
- $M_Z$, $M_H$, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, $\overline{m}_c$, $\overline{m}_b$, $m_t$
- Scale parameters for theoretical uncertainties
  $\Delta M_W$ (4 MeV), $\Delta\sin^2\theta^l_{eff}$ ($4.7\cdot10^{-5}$)

| | | |
|---|---|---|
| $M_H$ [GeV]$^{(\circ)}$ | $125.7 \pm 0.4$ | LHC |
| $M_W$ [GeV] | $80.385 \pm 0.015$ | Tevatron |
| $\Gamma_W$ [GeV] | $2.085 \pm 0.042$ | |
| $M_Z$ [GeV] | $91.1875 \pm 0.0021$ | LEP |
| $\Gamma_Z$ [GeV] | $2.4952 \pm 0.0023$ | |
| $\sigma^0_{\rm had}$ [nb] | $41.540 \pm 0.037$ | |
| $R^0_\ell$ | $20.767 \pm 0.025$ | |
| $A^{0,\ell}_{\rm FB}$ | $0.0171 \pm 0.0010$ | |
| $A_\ell$ $^{(\star)}$ | $0.1499 \pm 0.0018$ | SLC |
| $\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$ | $0.2324 \pm 0.0012$ | |
| $A_c$ | $0.670 \pm 0.027$ | SLC |
| $A_b$ | $0.923 \pm 0.020$ | |
| $A^{0,c}_{\rm FB}$ | $0.0707 \pm 0.0035$ | LEP |
| $A^{0,b}_{\rm FB}$ | $0.0992 \pm 0.0016$ | |
| $R^0_c$ | $0.1721 \pm 0.0030$ | |
| $R^0_b$ | $0.21629 \pm 0.00066$ | |
| $\overline{m}_c$ [GeV] | $1.27^{+0.07}_{-0.11}$ | |
| $\overline{m}_b$ [GeV] | $4.20^{+0.17}_{-0.07}$ | |
| $m_t$ [GeV] | $173.18 \pm 0.94$ | Tevatron |
| $\Delta\alpha^{(5)}_{\rm had}(M^2_Z)$ $^{(\triangle\bigtriangledown)}$ | $2757 \pm 10$ | |

# Global Fit: Results

**$\chi^2_{\text{min}}$/ndf = 21.8/14 → p-value = 0.08**

- large value of $\chi^2_{\text{min}}$ not due to inclusion of $M_H$ measurement
- without $M_H$ measurement:
  $\chi^2_{\text{min}}$ /ndf = 20.3/13 → naive p-value = 0.09



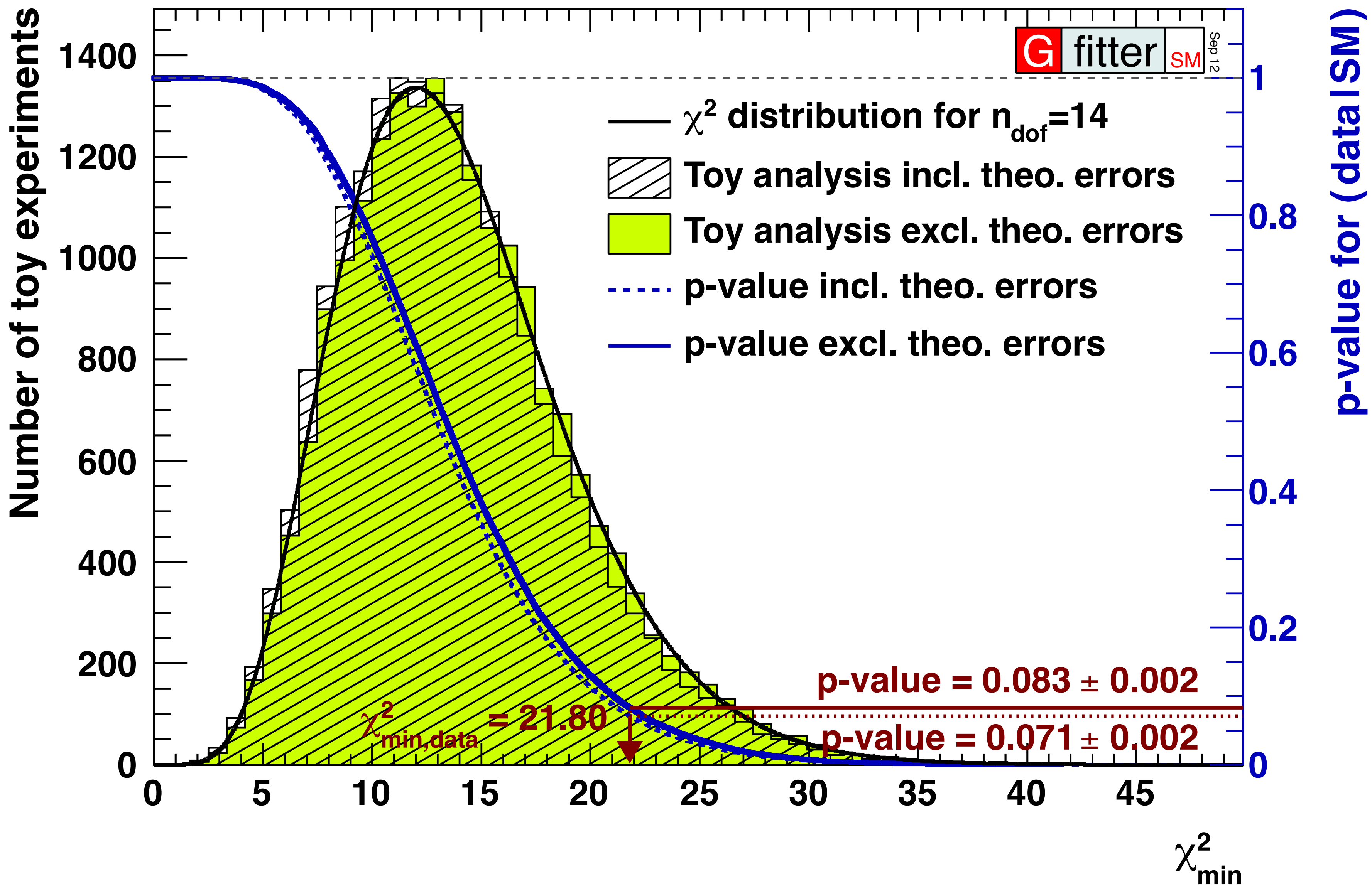
## Pull values after the fit

- Pull defined as $P = \dfrac{O_{\text{fit}} - O_{\text{meas}}}{\sigma_{\text{meas}}}$
- No pull value exceeds deviations of more than 3σ (good consistency of SM)
- Small values for $M_H$, $A_c$, $R^0_c$, $m_c$ and $m_b$ indicate that their input accuracies exceed the fit requirements
- Largest deviations in the b-sector: $A^{0,b}_{FB}$ and $R^0_b$ with 2.5σ and -2.4σ

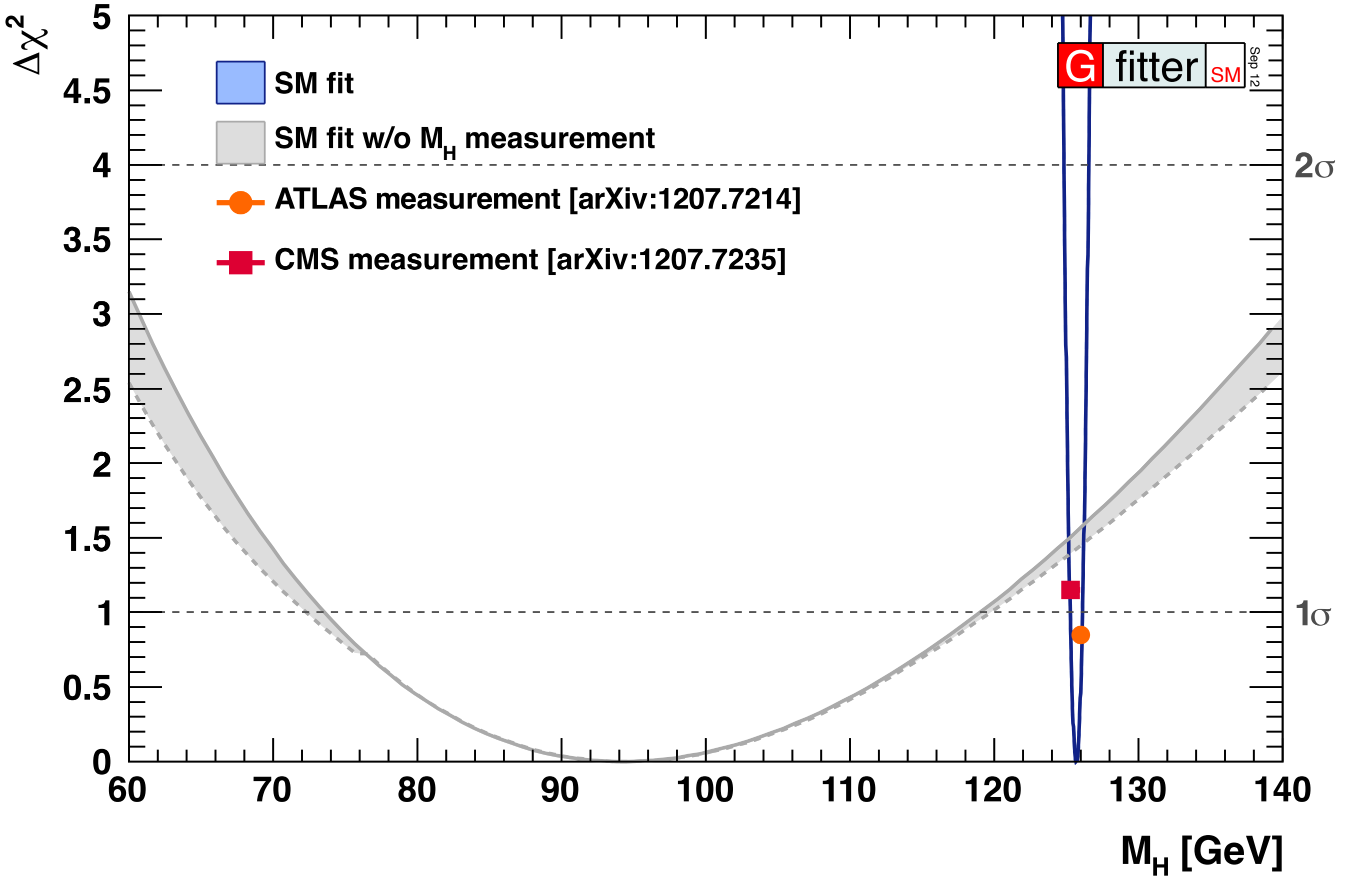
# Goodness of Fit

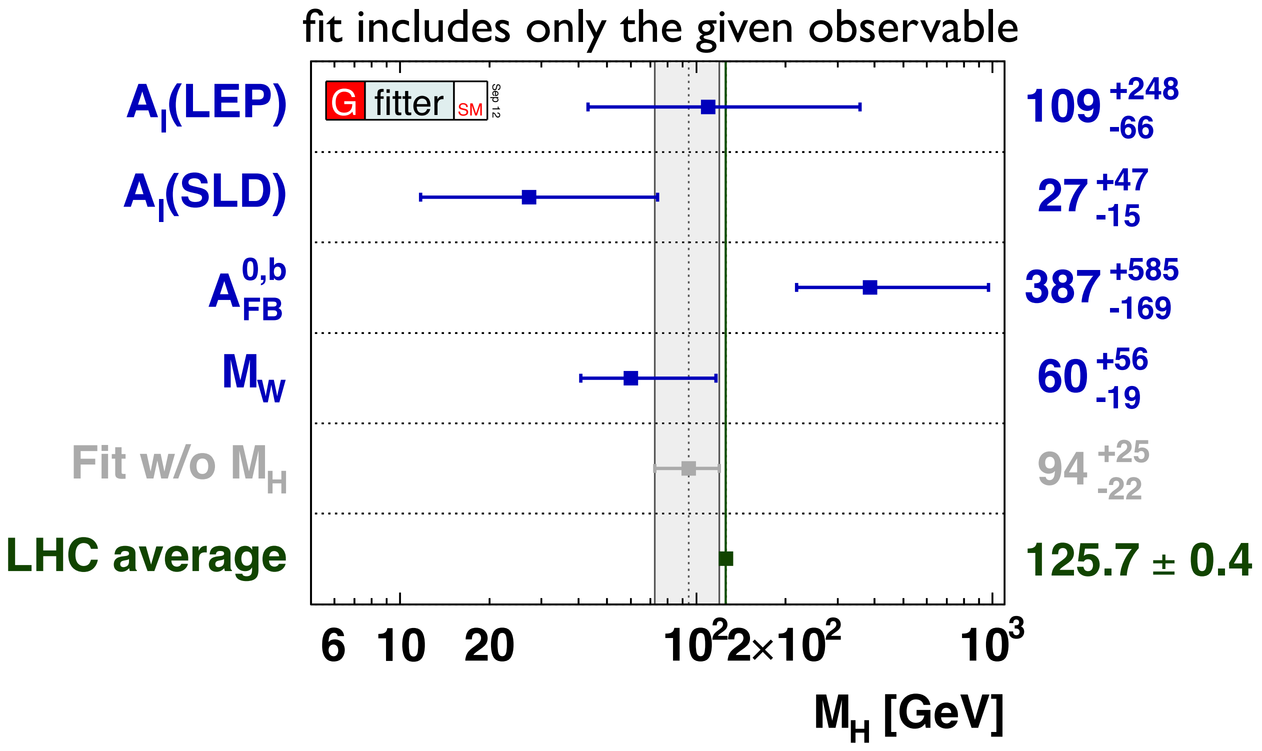
## Toy analysis with 20000 toy experiments

- p-value: probability for getting $\chi^2_{\text{min, toy}}$ larger than $\chi^2_{\text{min}}$ from data
- p-value: probability for wrongly rejecting the SM: 0.07 ± 0.01 (theo)



# Global Fit: Results





**Scan of the $\Delta\chi^2$ profile versus $M_H$**

- blue line: full SM fit
- grey band: fit without $M_H$ measurement
- fit without $M_H$ input gives $M_H = 94^{+25}_{-22}$ GeV
- consistent within 1.3σ with measurement

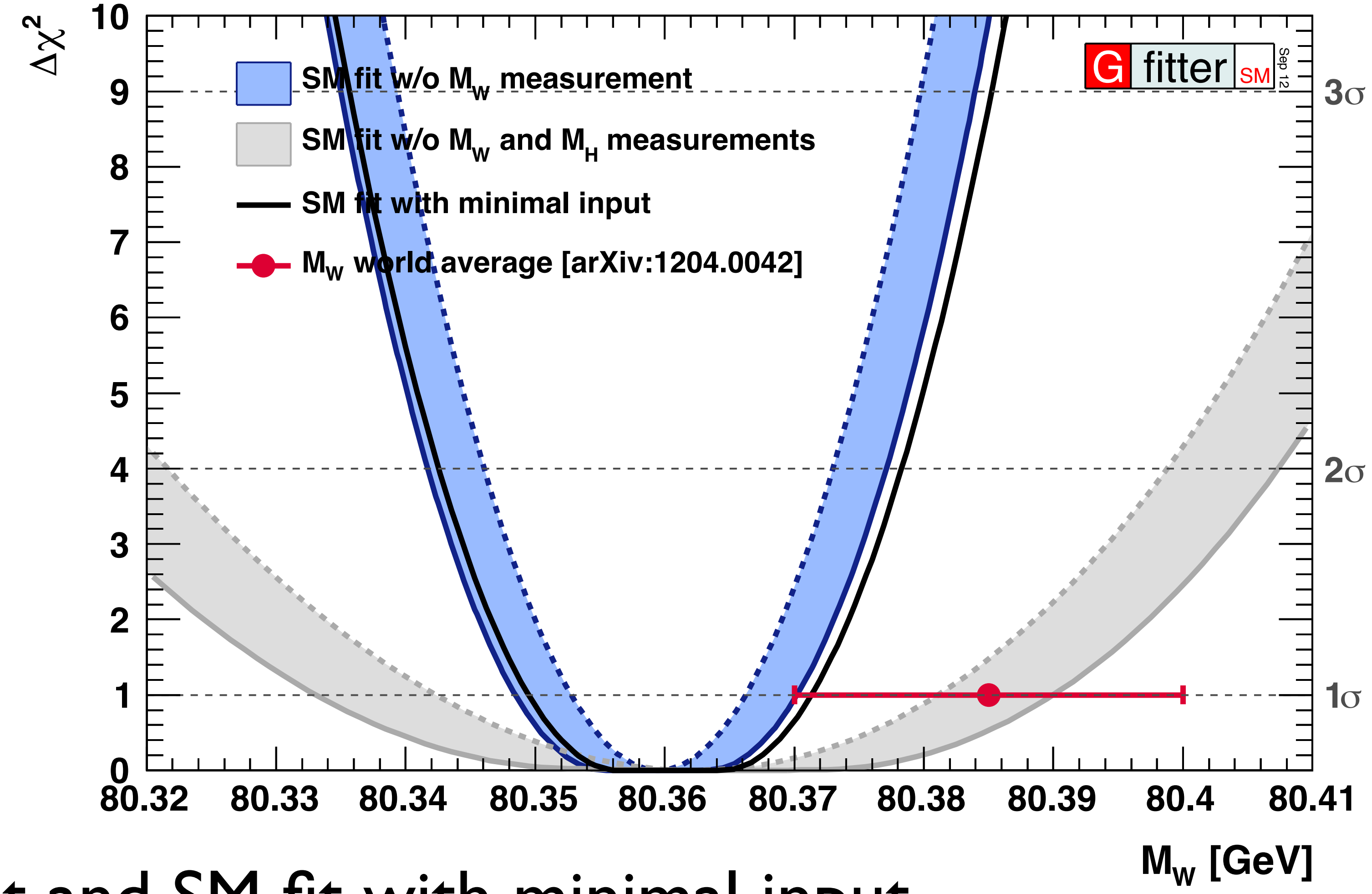
Determination of $M_H$ removing all sensitive observables except the given one:

Tension (2.5σ) between $A^{0,b}_{FB}$, $A_{lep}$(SLD) and $M_W$ visible

# Indirect Determination: W Mass

**Scan of the $\Delta\chi^2$ profile versus $M_W$**

- $M_H$ measurement allows for precise constraint of $M_W$
- also shown: SM fit with minimal input: $M_Z$, $G_F$, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, $M_H$, $m_c$, $m_b$, $m_t$



- Consistency between total fit and SM fit with minimal input
- Fit result for the indirect determination of $M_W$:

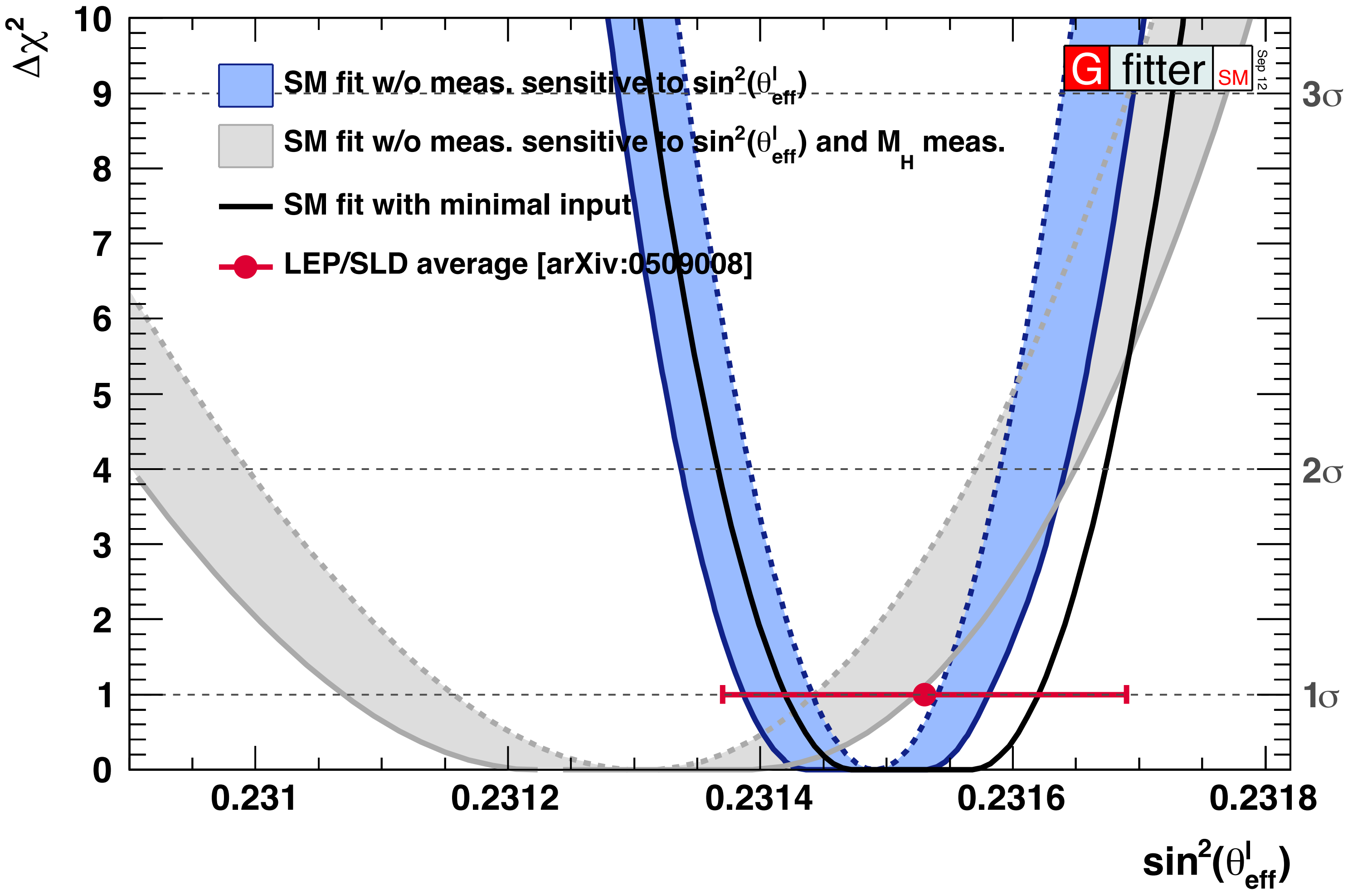
$$\begin{aligned} M_W &= 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \\ &\quad \pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm 0.0040_{\rm theo}\,, \\ &= 80.359 \pm 0.011_{\rm tot}\,, \end{aligned}$$

More precise than the direct measurements

# The Effective Weak Mixing

**Scan of the $\Delta\chi^2$ profile versus $\sin^2\theta^l_{eff}$**

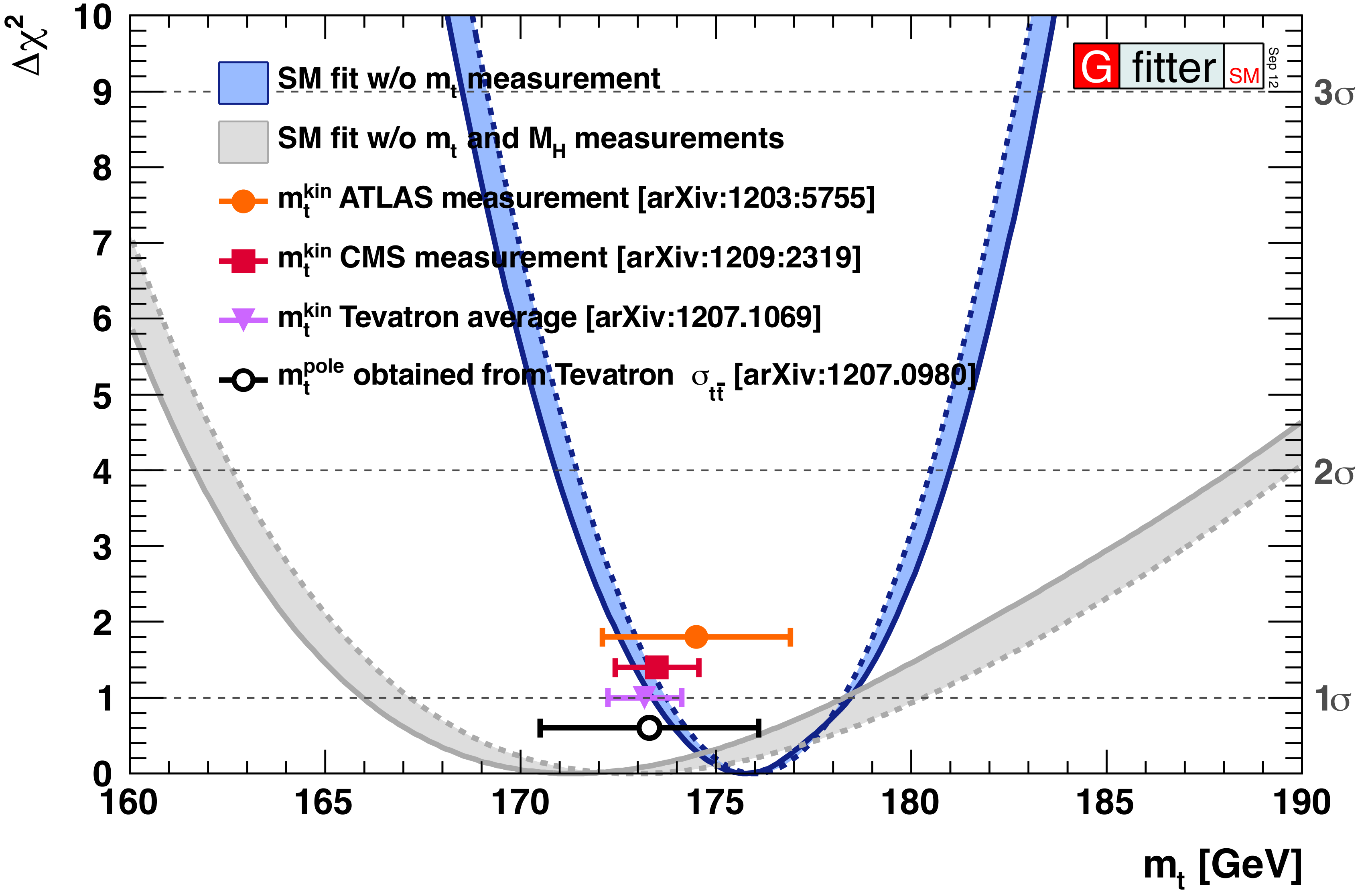
- all observables sensitive to $\sin^2\theta^l_{eff}$ removed from fit
- $M_H$ measurement allows for precise constraint of $\sin^2\theta^l_{eff}$
- also shown: SM fit with minimal input



$$
\begin{aligned}
\sin^2\theta^\ell_{\rm eff} \quad &= \quad 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \\
&\qquad \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{\rm theo}\,, \\
&= \quad 0.23150 \pm 0.00010_{\rm tot}\,,
\end{aligned}
$$

More precise than the direct determination from LEP/SLD measurements

# Indirect Determination: Top Mass

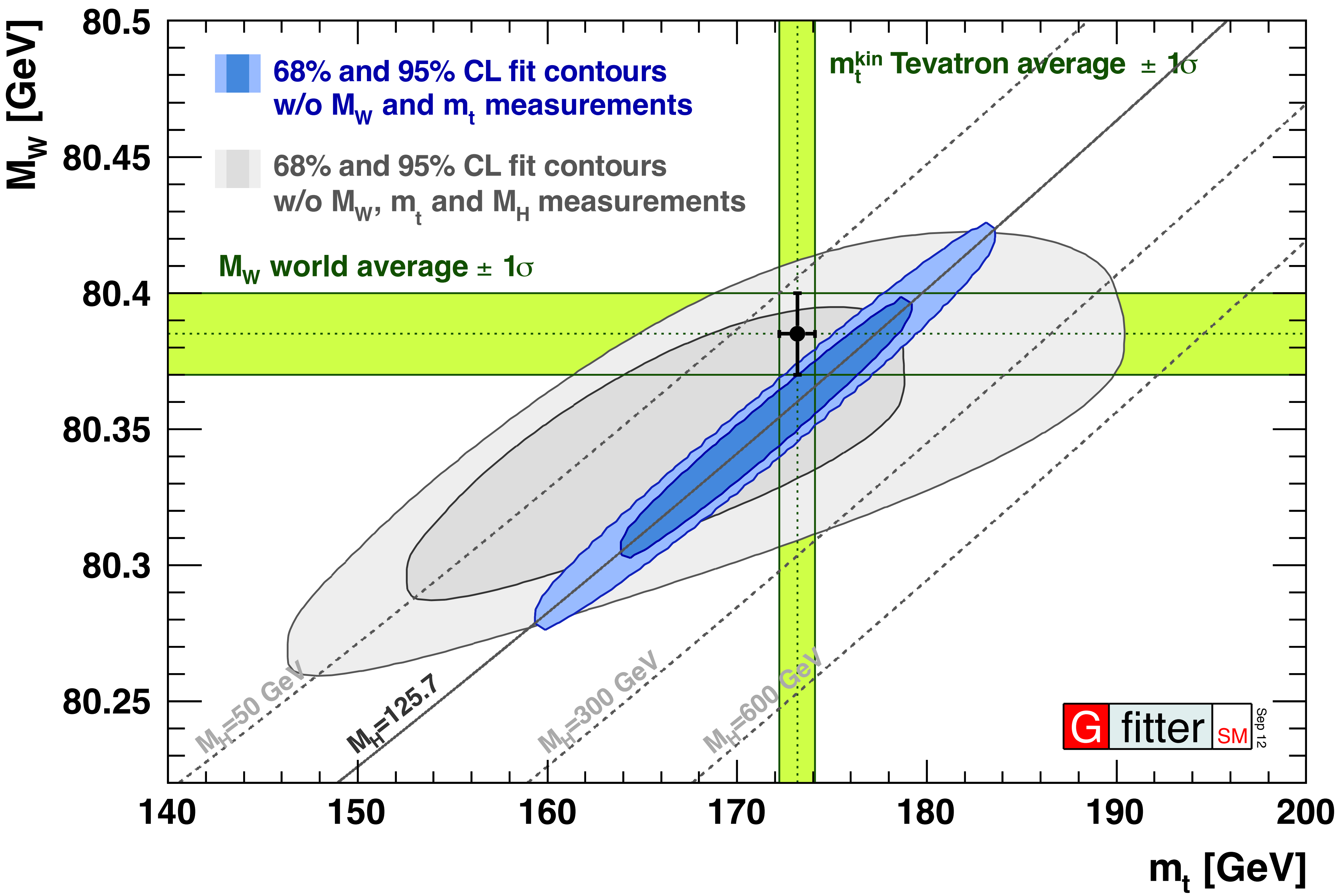


## Scan of the $\Delta\chi^2$ profile versus $m_t$

- consistency with direct measurements
- $M_H$ measurement allows for better constraint of $m_t$

$m_t = 175.8\,^{+2.7}_{-2.4}\,\mathrm{GeV}$ (Tevatron average: $m_t$ = 173.2 ± 0.9 GeV)

# W and Top Mass



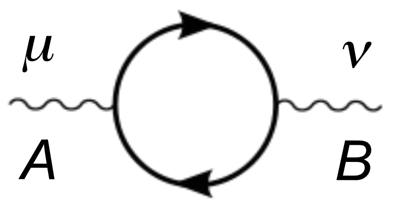
**68% and 95% CL contours of fit without using $M_W$, $m_t$ (and $M_H$)**

- Impressive consistency of the SM

# Beyond the SM

At low energies, BSM physics appears dominantly through vacuum polarisation

- Aka, *oblique corrections*



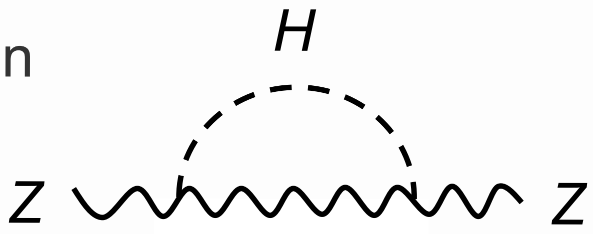
$$= i\Pi^{\mu\nu}_{AB=\{W,Z,\gamma\}}(q)$$

- Direct corrections (vertex, box, bremsstrahlung) generally suppressed by $m_f / \Lambda$

Oblique corrections reabsorbed into electroweak parameters $\Delta\rho$, $\Delta\kappa$, $\Delta r$

Electroweak fit sensitive to BSM physics through oblique corrections

- In direct competition with Higgs loop corrections



- Oblique corrections from New Physics described through ***STU* parameters**

  [Peskin-Takeuchi, Phys. Rev. D46, 381 (1992)]

$$\boxed{O_{\text{meas}} = O_{\text{SM,ref}}(M_H, m_t) + c_S S + c_T T + c_U U}$$

| | |
|---|---|
| ***S*** : | (*S*+*U*) New Physics contributions to neutral (charged) currents |
| ***T*** : | Difference between neutral and charged current processes – sensitive to weak isospin violation |
| ***U*** : | Constrained by $M_W$ and $\Gamma_W$. Usually very small in NP models (often: *U*=0) |

- Also considered: correction to $Z \to bb$ coupling, and extended parameters (*VWX*)

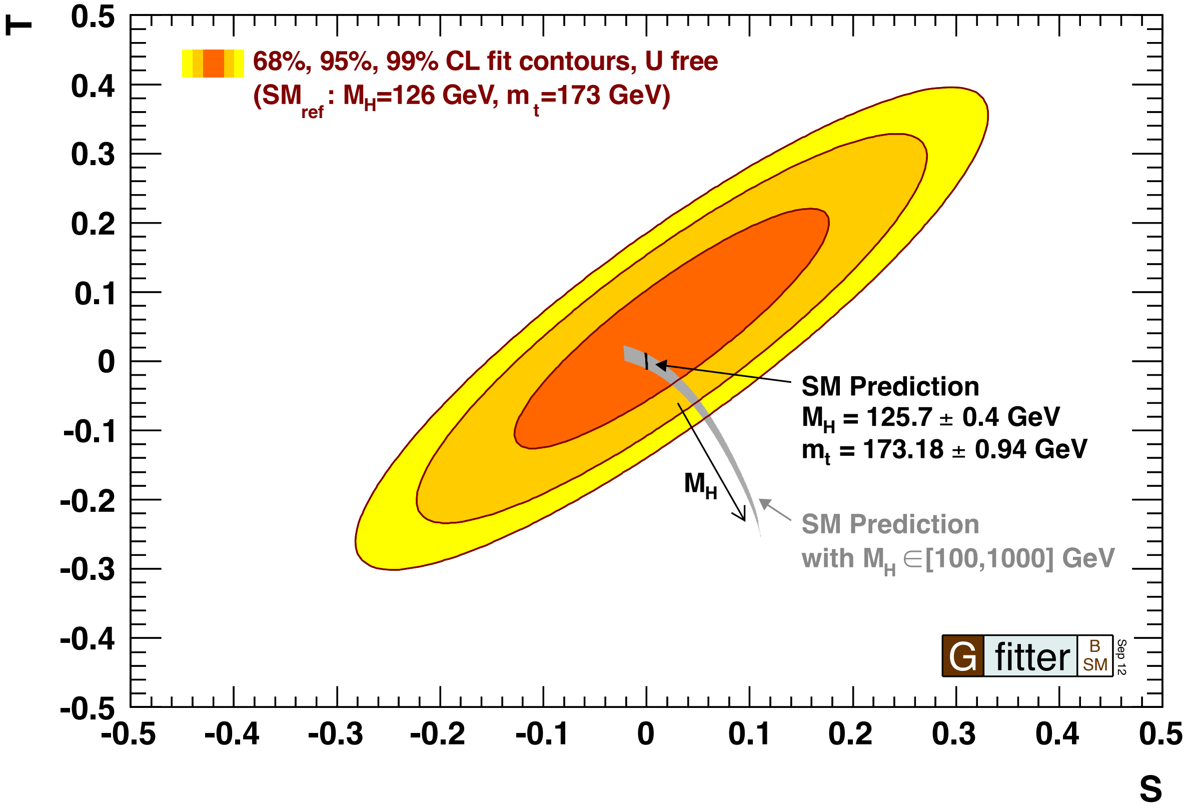
  [Burgess et al., PLB 326, 276 (1994), PRD 49, 6115 (1994)]

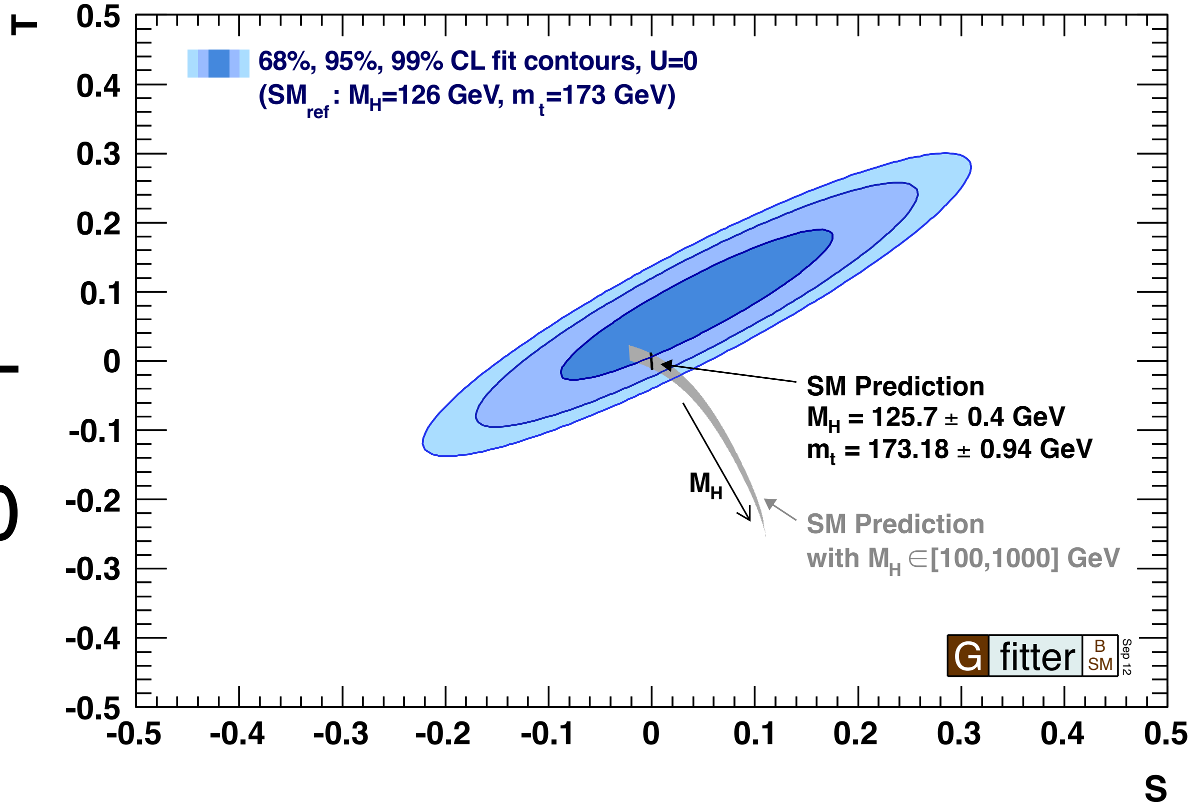
# Constraints on S, T and U

## S, T, U obtained by fit to EW observables

- SM reference chosen to be  
  $M_{H,ref}$ = 126 GeV  
  $m_{t,ref}$ = 173 GeV
  - this defines (0, 0, 0)
  - S,T depend logarithmically on $M_H$
- Fit result:  
  S = 0.03 ± 0.10  
  T = 0.05 ± 0.12  
  U = 0.03 ± 0.10  
  with large correlation between S and T
- Stronger constraints from fit with U=0

No indication of new physics





# Summary

**Assuming the newly discovered boson is the SM Higgs**

- all fundamental parameters of the SM are known
- possibility to overconstrain the SM at the electroweak scale
- global EW fit has been redone, with a p-value of 0.07
- small p-value comes mostly from $R^0_b$ and $A^{0,b}_{FB}$

**Knowledge of $M_H$ allows for precision determinations of**

- W mass, top mass, $\sin^2\theta^l_{eff}$
- detailed information in arXiv:1209.2716 and recent updates on [www.cern.ch/gfitter](www.cern.ch/gfitter)

**EW Fit allows to constrain many BSM models**

- no signs of new physics from oblique parameters
- stay tuned for more results

# Additional Material

| Parameter | Input value | Free in fit | Fit result incl. $M_H$ | Fit result not incl. $M_H$ | Fit result incl. $M_H$ but not exp. input in row |
|---|---|---|---|---|---|
| $M_H$ [GeV]$^{(\circ)}$ | $125.7 \pm 0.4$ | yes | $125.7 \pm 0.4$ | $94\,^{+25}_{-22}$ | $94\,^{+25}_{-22}$ |
| $M_W$ [GeV] | $80.385 \pm 0.015$ | – | $80.367 \pm 0.007$ | $80.380 \pm 0.012$ | $80.359 \pm 0.011$ |
| $\Gamma_W$ [GeV] | $2.085 \pm 0.042$ | – | $2.091 \pm 0.001$ | $2.092 \pm 0.001$ | $2.091 \pm 0.001$ |
| $M_Z$ [GeV] | $91.1875 \pm 0.0021$ | yes | $91.1878 \pm 0.0021$ | $91.1874 \pm 0.0021$ | $91.1983 \pm 0.0116$ |
| $\Gamma_Z$ [GeV] | $2.4952 \pm 0.0023$ | – | $2.4954 \pm 0.0014$ | $2.4958 \pm 0.0015$ | $2.4951 \pm 0.0017$ |
| $\sigma^0_{\rm had}$ [nb] | $41.540 \pm 0.037$ | – | $41.479 \pm 0.014$ | $41.478 \pm 0.014$ | $41.470 \pm 0.015$ |
| $R^0_\ell$ | $20.767 \pm 0.025$ | – | $20.740 \pm 0.017$ | $20.743 \pm 0.018$ | $20.716 \pm 0.026$ |
| $A^{0,\ell}_{\rm FB}$ | $0.0171 \pm 0.0010$ | – | $0.01627 \pm 0.0002$ | $0.01637 \pm 0.0002$ | $0.01624 \pm 0.0002$ |
| $A_\ell\ ^{(\star)}$ | $0.1499 \pm 0.0018$ | – | $0.1473\,^{+0.0006}_{-0.0008}$ | $0.1477 \pm 0.0009$ | $0.1468 \pm 0.0005^{(\dagger)}$ |
| $\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$ | $0.2324 \pm 0.0012$ | – | $0.23148\,^{+0.00011}_{-0.00007}$ | $0.23143\,^{+0.00010}_{-0.00012}$ | $0.23150 \pm 0.00009$ |
| $A_c$ | $0.670 \pm 0.027$ | – | $0.6680\,^{+0.00025}_{-0.00038}$ | $0.6682\,^{+0.00042}_{-0.00035}$ | $0.6680 \pm 0.00031$ |
| $A_b$ | $0.923 \pm 0.020$ | – | $0.93464\,^{+0.00004}_{-0.00007}$ | $0.93468 \pm 0.00008$ | $0.93463 \pm 0.00006$ |
| $A^{0,c}_{\rm FB}$ | $0.0707 \pm 0.0035$ | – | $0.0739\,^{+0.0003}_{-0.0005}$ | $0.0740 \pm 0.0005$ | $0.0738 \pm 0.0004$ |
| $A^{0,b}_{\rm FB}$ | $0.0992 \pm 0.0016$ | – | $0.1032\,^{+0.0004}_{-0.0006}$ | $0.1036 \pm 0.0007$ | $0.1034 \pm 0.0004$ |
| $R^0_c$ | $0.1721 \pm 0.0030$ | – | $0.17223 \pm 0.00006$ | $0.17223 \pm 0.00006$ | $0.17223 \pm 0.00006$ |
| $R^0_b$ | $0.21629 \pm 0.00066$ | – | $0.21474 \pm 0.00003$ | $0.21475 \pm 0.00003$ | $0.21473 \pm 0.00003$ |
| $\overline{m}_c$ [GeV] | $1.27\,^{+0.07}_{-0.11}$ | yes | $1.27\,^{+0.07}_{-0.11}$ | $1.27\,^{+0.07}_{-0.11}$ | – |
| $\overline{m}_b$ [GeV] | $4.20\,^{+0.17}_{-0.07}$ | yes | $4.20\,^{+0.17}_{-0.07}$ | $4.20\,^{+0.17}_{-0.07}$ | – |
| $m_t$ [GeV] | $173.18 \pm 0.94$ | yes | $173.52 \pm 0.88$ | $173.14 \pm 0.93$ | $175.8\,^{+2.7}_{-2.4}$ |
| $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\ ^{(\triangle\bigtriangledown)}$ | $2757 \pm 10$ | yes | $2755 \pm 11$ | $2757 \pm 11$ | $2716\,^{+49}_{-43}$ |
| $\alpha_s(M_Z^2)$ | – | yes | $0.1191 \pm 0.0028$ | $0.1192 \pm 0.0028$ | $0.1191 \pm 0.0028$ |
| $\delta_{\rm th} M_W$ [MeV] | $[-4, 4]_{\rm theo}$ | yes | $4$ | $4$ | – |
| $\delta_{\rm th} \sin^2\theta^\ell_{\rm eff}\ ^{(\triangle)}$ | $[-4.7, 4.7]_{\rm theo}$ | yes | $-1.4$ | $4.7$ | – |

# New Calculation of $R^0_b$

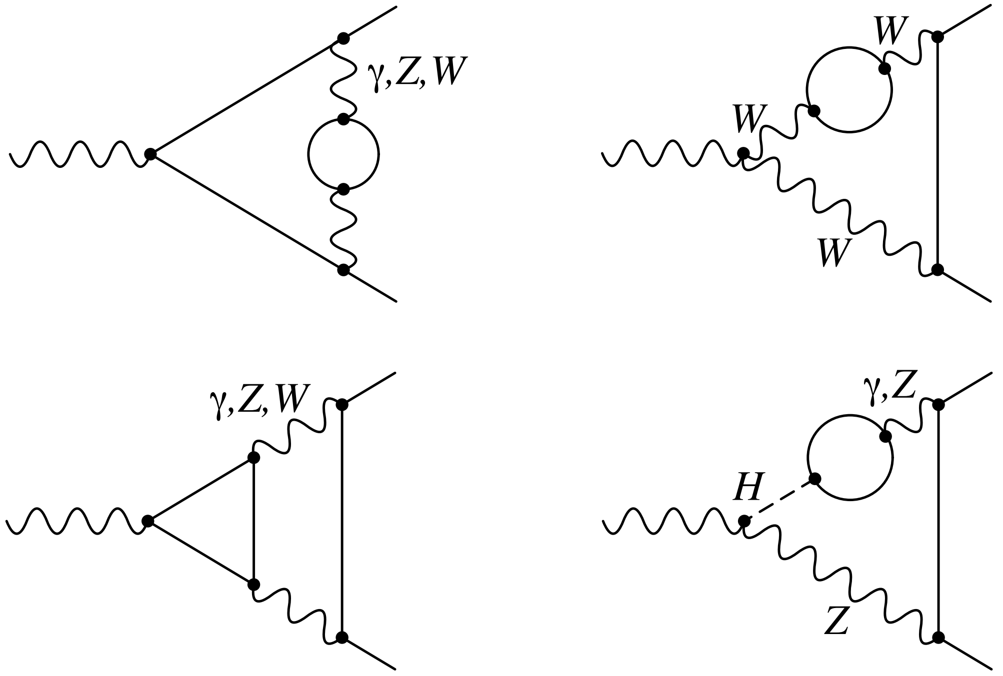
[A. Freitas et al., JHEP 1208, 050 (2012)]

## Full two-loop calculation of Z→$b\bar{b}$

- The branching ratio $R^0_b$:
  partial decay width of Z→$b\bar{b}$ and Z→$q\bar{q}$

$$R_b \equiv \frac{\Gamma_b}{\Gamma_{\rm had}} = \frac{\Gamma_b}{\Gamma_d + \Gamma_u + \Gamma_s + \Gamma_c + \Gamma_b}$$

- Two-loop corrections are rather large compared to the one-loop results

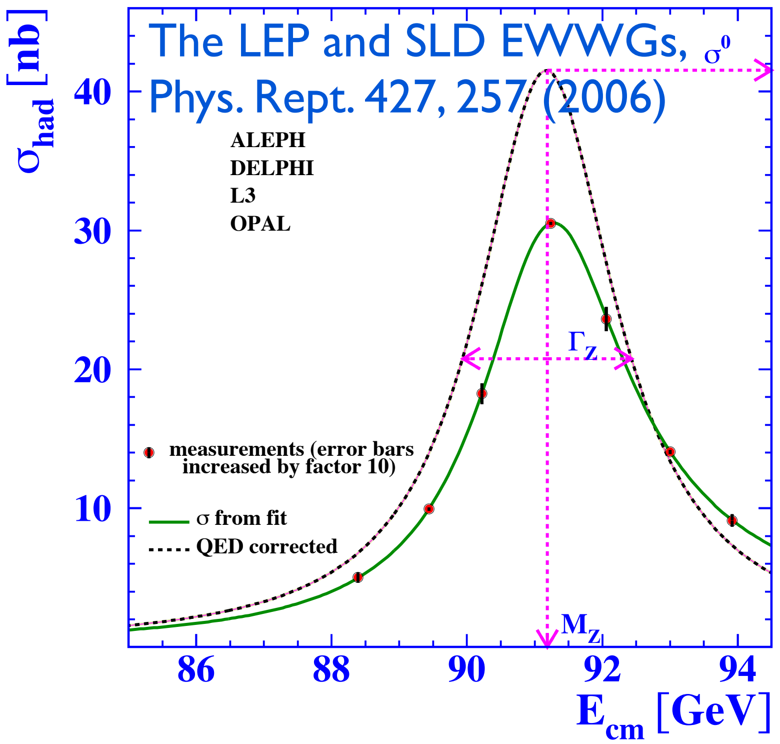


fermionic EW two-loop corrections to the vertex form factors

| | 1-loop EW and QCD correction to FSR | 2-loop EW correction | 2-loop EW and 2+3-loop QCD correction to FSR | 1+2-loop QCD correction to gauge boson selfenergies |
|---|---|---|---|---|
| $M_{\rm H}$ [GeV] | $\mathcal{O}(\alpha) + {\rm FSR}_{1-{\rm loop}}$ $[10^{-3}]$ | $\mathcal{O}(\alpha^2_{\rm ferm})$ $[10^{-4}]$ | $\mathcal{O}(\alpha^2_{\rm ferm}) + {\rm FSR}_{>1-{\rm loop}}$ $[10^{-4}]$ | $\mathcal{O}(\alpha\alpha_{\rm s}, \alpha\alpha^2_{\rm s})$ $[10^{-4}]$ |
| 100 | −3.632 | −6.569 | −9.333 | −0.404 |
| 200 | −3.651 | −6.573 | −9.332 | −0.404 |
| 400 | −3.675 | −6.581 | −9.331 | −0.404 |

# $\alpha_s(M_Z)$ from Z→hadrons

- Fit of electroweak precision observables
- Input mostly from LEP data from the Z-peak
- Determination of $\alpha_s$: most sensitivity through total hadronic cross section at the Z-pole and the partial leptonic width


The LEP and SLD EWWGs, Phys. Rept. 427, 257 (2006)

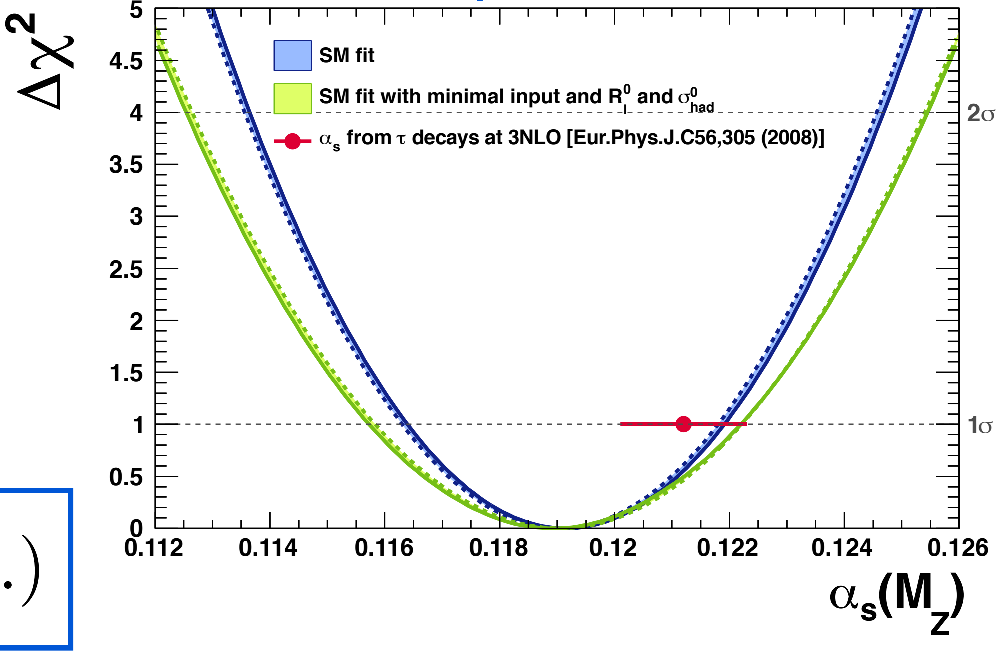
$$\sigma^0_{\rm had} \equiv \frac{12\pi}{m_{\rm Z}^2}\frac{\Gamma_{\rm ee}\Gamma_{\rm had}}{\Gamma_{\rm Z}^2} \qquad R^0_\ell \equiv \Gamma_{\rm had}/\Gamma_{\ell\ell}$$

obtained from the four LEP experiments, 17 million Z decays

Complete $O(\alpha_s^4)$ calculation available:

[P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)]

Gfitter Group, arXiv:1209.2716



$$\boxed{\alpha_s(M_Z) = 0.1191 \pm 0.0028\,(\text{exp.}) \pm 0.0001\,(\text{theo.})}$$

Improvement in precision only with ILC/GigaZ expected