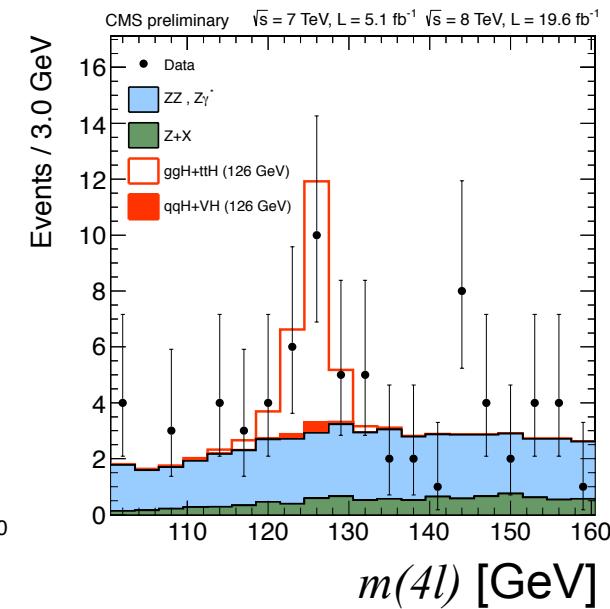
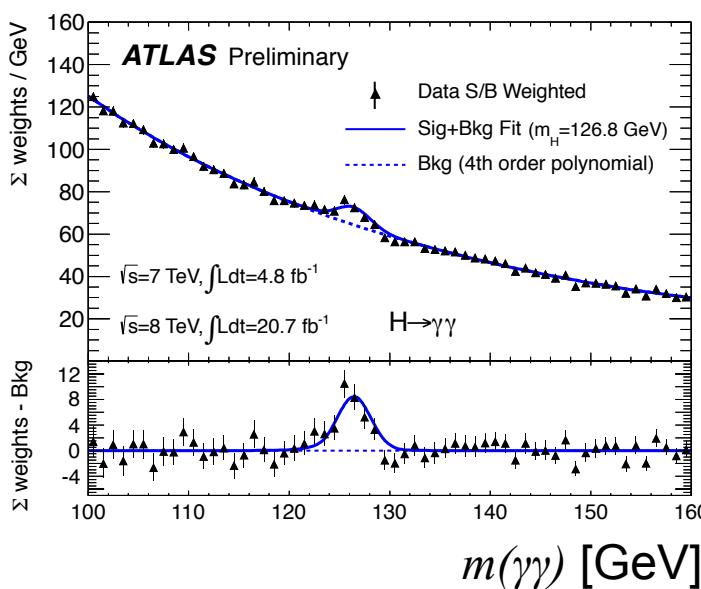




<http://cern.ch/Gfitter>

EPJC 72, 2205 (2012), arXiv:1209.2716

## *The ElectroWeak fit of Standard Model after the Discovery of the Higgs-like boson*



# Gfitter

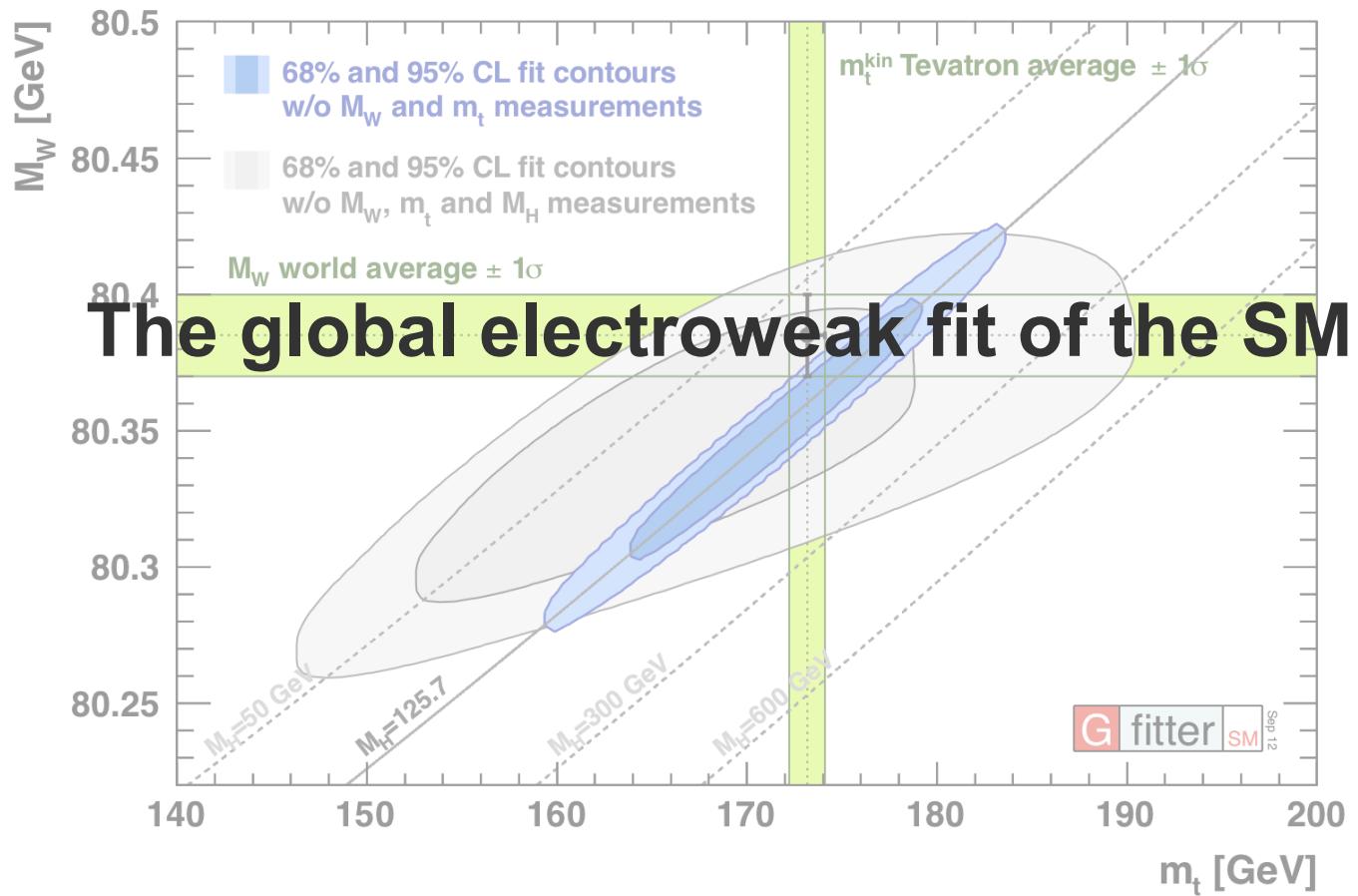
This presentation:

- Introduction to Gfitter
- Introduction to the electroweak fit of the Standard Model
- Inputs to the electroweak fit
- Fit results
- Future Prospects of ILC
- Conclusion & Outlook



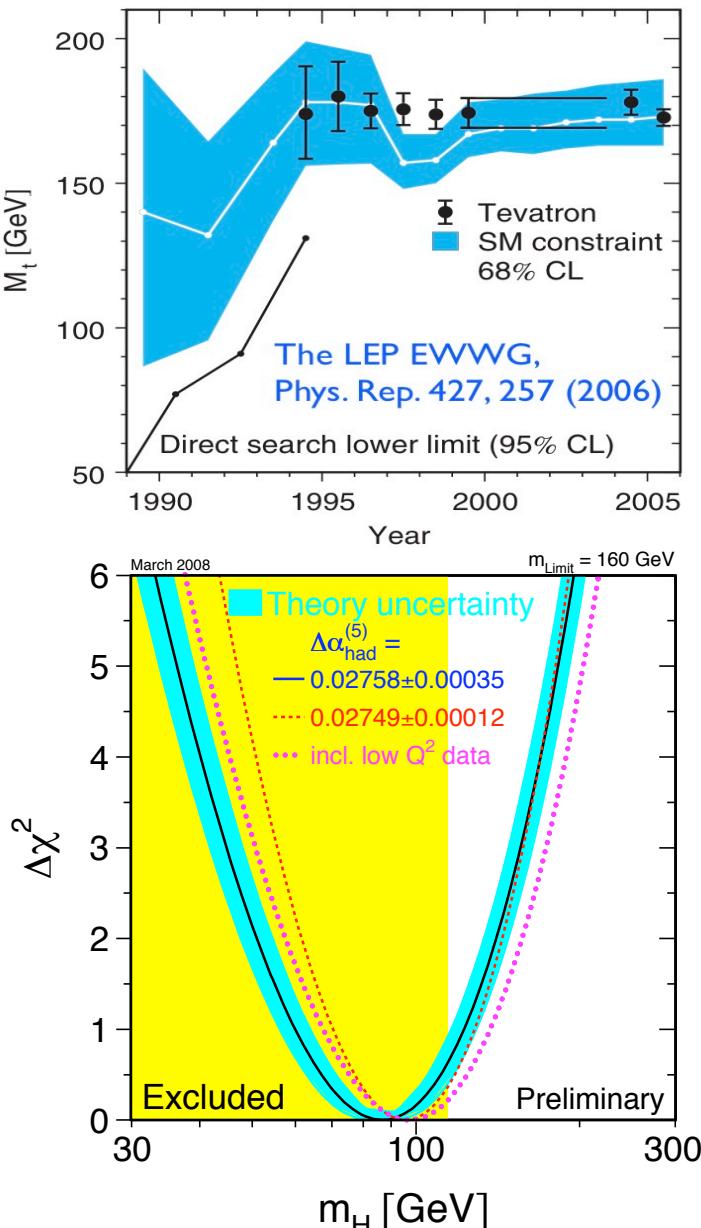
## A Generic Fitter Project for HEP Model Testing

- Gfitter = state-of-the-art HEP model testing tool for LHC era
- Gfitter software and features:
  - Modular, object-oriented C++, relying on ROOT, XML, python, etc.
  - Core package with data-handling, fitting, and statistics tools
    - Various fitting tools: Minuit (1/2), Genetic Algorithms, Simulated Annealing, etc.
    - Consistent treatment of statistical, systematic, theoretical uncertainties (Rfit prescription), correlations, and inter-parameter dependencies.
      - » Theoretical uncertainties included in  $\chi^2$  with flat likelihood in allowed ranges
    - Full statistics analysis: goodness-of-fit, p-values, parameter scans, MC analyses.
  - Independent “plug-in” physics libraries: SM, 2HDM, multiple BSM model, ...
- Our publications and new results available at: [www.cern.ch/Gfitter](http://www.cern.ch/Gfitter)



# Global EW fits: a long history

- Huge amount of pioneering work by many!
  - Needed to understand importance of loop corrections
    - Observables (now) known at least at two-loop order, sometimes more.
  - High-precision Standard Model (SM) predictions and measurements required
    - First from LEP/SLC, then Tevatron, now LHC.
- EW fits performed by many groups in past
  - D. Bardinet al. (ZFITTER), G. Passarino et al. (TOPAZ0), LEP EW WG (M. Grünwald, K. Mönig et al.), J. Erler (GAPP), ...
  - Important results obtained!
- (Global SM fits also used at lower energies [CKM-matrix], and many groups pursuing global beyond-SM fits.)

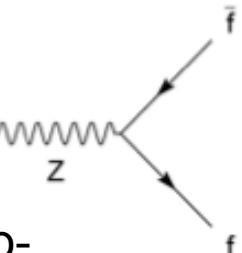


# The predictive power of the SM

- As the Z boson couples to all fermions, it is ideal to measure & study both the electroweak and strong interactions.

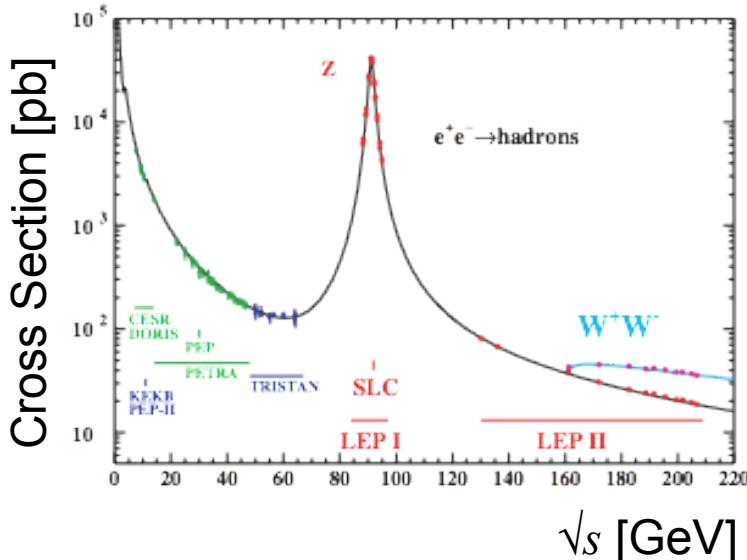
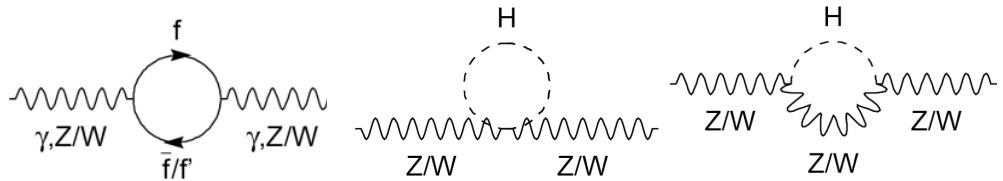
- Tree level relations for  $Z \rightarrow f\bar{f}$

$$i\bar{f}\gamma^\mu (g_{V,f} - g_{A,f}\gamma_5) f Z_\mu$$



- Unification connects the electromagnetic and weak couplings

- The impact of radiative corrections
  - Absorbed into EW form factors:  $\rho$ ,  $\kappa$ ,  $\Delta r$
  - Effective couplings at the Z-pole
  - Quadratically dependent on  $m_t$ , *logarithmic* dependence on  $M_H$



$$g_{V,f} = \sqrt{\rho_Z^f} \left( I_3^f - 2Q^f \sin^2 \theta_{\text{eff}}^f \right)$$

$$g_{A,f} = \sqrt{\rho_Z^f} I_3^f$$

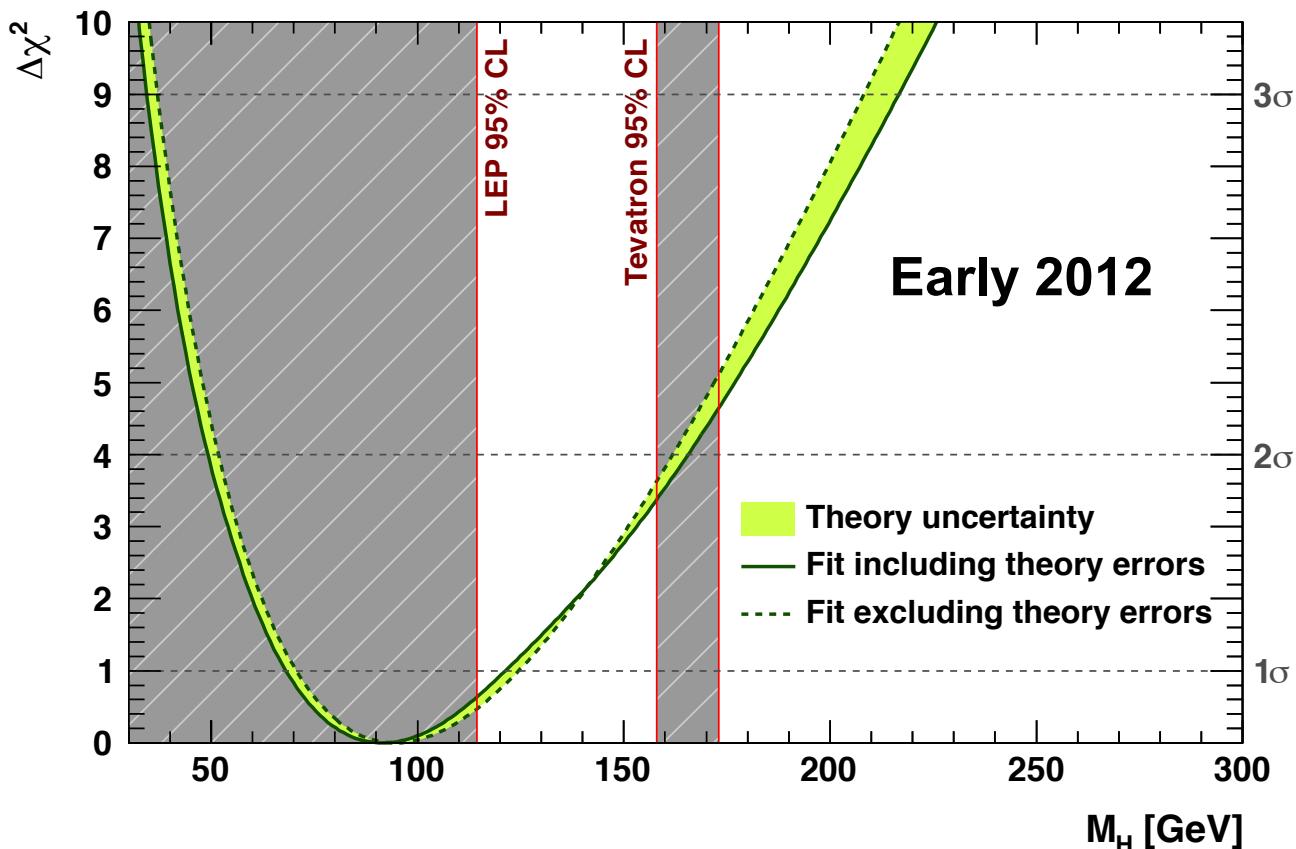
$$\sin^2 \theta_{\text{eff}}^f = \kappa_Z^f \sin^2 \theta_W$$

$$M_W^2 = \frac{M_Z^2}{2} \left( 1 + \sqrt{1 - \frac{\sqrt{8\pi\alpha}(1 + \Delta r)}{G_F M_Z^2}} \right)$$

# Hunt for the Higgs



Gfitter group, EPJC 72, 2003 (2012)

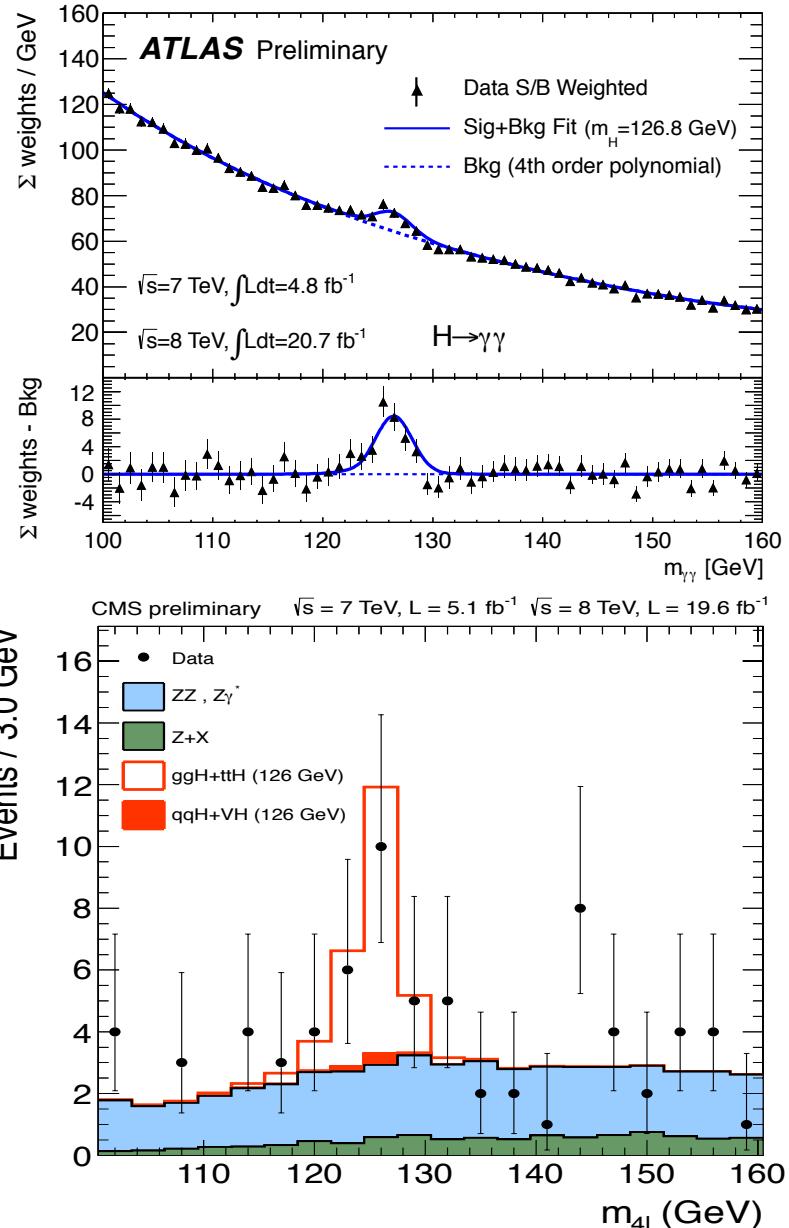


- $M_H$  was last missing input parameter of the electroweak fit
- Indirect determination from EW fit (2012):  $M_H = 96^{+31}_{-24}$  GeV
- (Direct Higgs limits of course also available in the EW fit.)

# The SM fit with Gfitter, including the Higgs



- Discovery of Higgs-like boson at LHC
  - Cross section, branching ratios, spin, parity sofar compatible with SM Higgs boson.
  - This talk: assume boson is SM Higgs.
  - Use in EW fit:  $M_H = 125.7 \pm 0.4$  GeV
    - ATLAS:  $M_H = 126.0 \pm 0.4 \pm 0.4$  GeV
    - CMS:  $M_H = 125.3 \pm 0.4 \pm 0.5$  GeV  
[arXiv:1207.7214, arXiv:1207.7235]
  - Change in average between fully uncorrelated and fully correlated systematic uncertainties is minor:  
 $\delta M_H : 0.4 \rightarrow 0.5$  GeV
- Unique situation: for first time SM is fully over-constrained  
→ Test its self-consistency!
- The focus of this talk ...



# Measurements at the Z-pole (1/2)

- Total cross-section
  - Express in terms of partial decay width of initial and final width:

$$\sigma_{f\bar{f}}^Z = \sigma_{f\bar{f}}^0 \frac{s \Gamma_Z^2}{(s - M_Z^2)^2 + s^2 \Gamma_Z^2 / M_Z^2} \frac{1}{R_{\text{QED}}} \quad \text{with} \quad \sigma_{f\bar{f}}^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

Corrected for QED radiation

- Full width:  $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{had}} + \Gamma_{\text{inv}}$
- (Correlated set of measurements.)

- Set of input (width) parameters to EW fit:

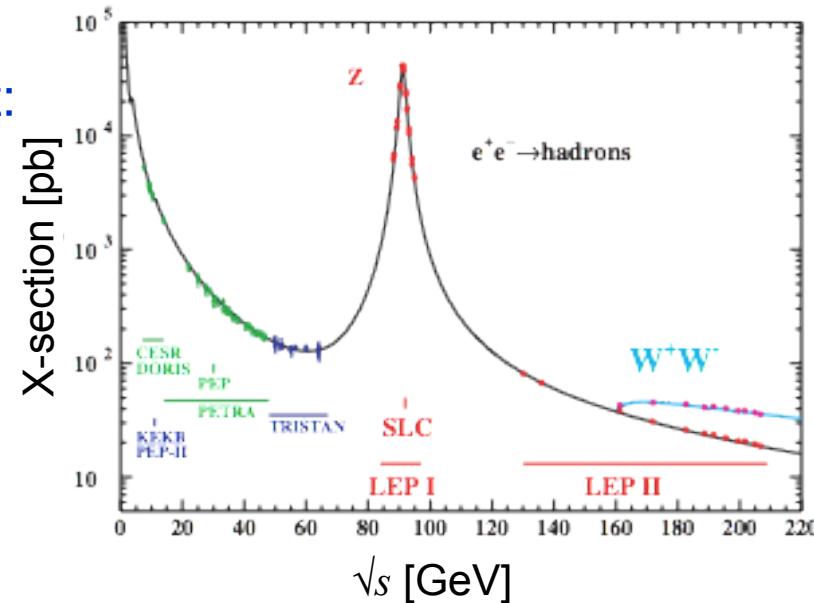
- Z mass and width:  $M_Z$ ,  $\Gamma_Z$
- Hadronic pole cross section:

$$\sigma_{\text{had}}^0 = 12\pi/M_Z^2 \cdot \Gamma_{ee}\Gamma_{\text{had}}/\Gamma_Z^2$$

- Three leptonic ratios (lepton univ.):

$$R_\ell^0 = R_e^0 = \Gamma_{\text{had}}/\Gamma_{ee} \quad (= R_\mu^0 = R_\tau^0)$$

- Hadronic width ratios:  $R_b^0, R_c^0$



# Measurements at the Z-pole (2/2)

## ■ Definition of Asymmetry

- Distinguish vector and axial-vector couplings of the Z

$$A_f = \frac{g_{L,f}^2 - g_{R,f}^2}{g_{L,f}^2 + g_{R,f}^2} = \frac{2g_{V,f} g_{A,f}}{g_{V,f}^2 + g_{A,f}^2}$$

- Directly related to  $\sin^2 \theta_{\text{eff}}^{f\bar{f}} = \frac{1}{4Q_f} \left( 1 + \mathcal{R}e \left( \frac{g_{V,f}}{g_{A,f}} \right) \right)$

## ■ Observables

- In case of no beam polarisation (LEP)  
use final state angular distribution to  
define *forward/backward asymmetry*

$$A_{FB}^f = \frac{N_F^f - N_B^f}{N_F^f + N_B^f}$$

$$A_{FB}^{0,f} = \frac{3}{4} A_e A_f$$

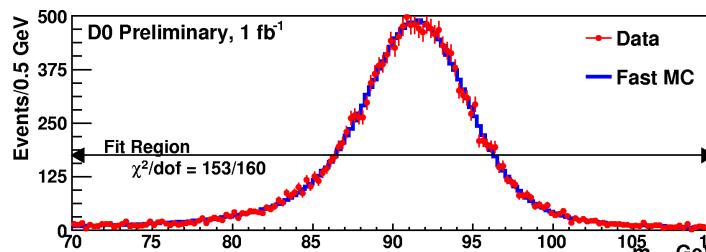
- Polarised beams (SLC),  
define *left/right asymmetry*

$$A_{LR}^f = \frac{N_L^f - N_R^f}{N_L^f + N_R^f} \frac{1}{\langle |P|_e \rangle} \quad A_{LR}^0 = A_e$$

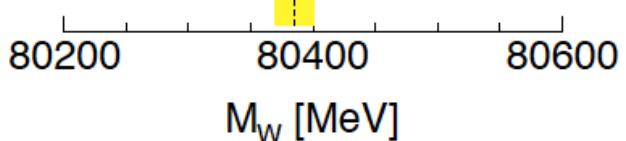
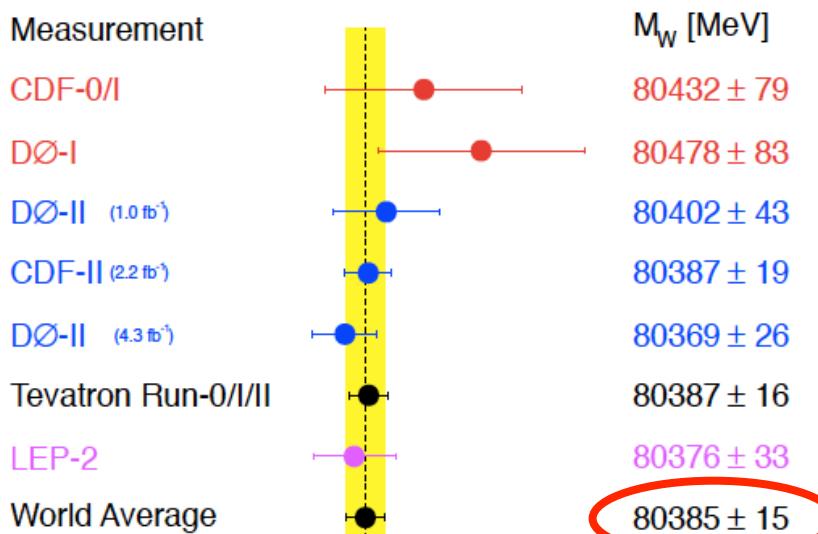
- Measurements:  $A_{FB}^{0,\ell}$ ,  $A_{FB}^{0,c}$ ,  $A_{FB}^{0,b}$ ,  $A_\ell$ ,  $A_c$ ,  $A_b$

# 2012 averages for $M_W$ and $m_{top}$

Latest Tevatron result from: arXiv:1204.0042

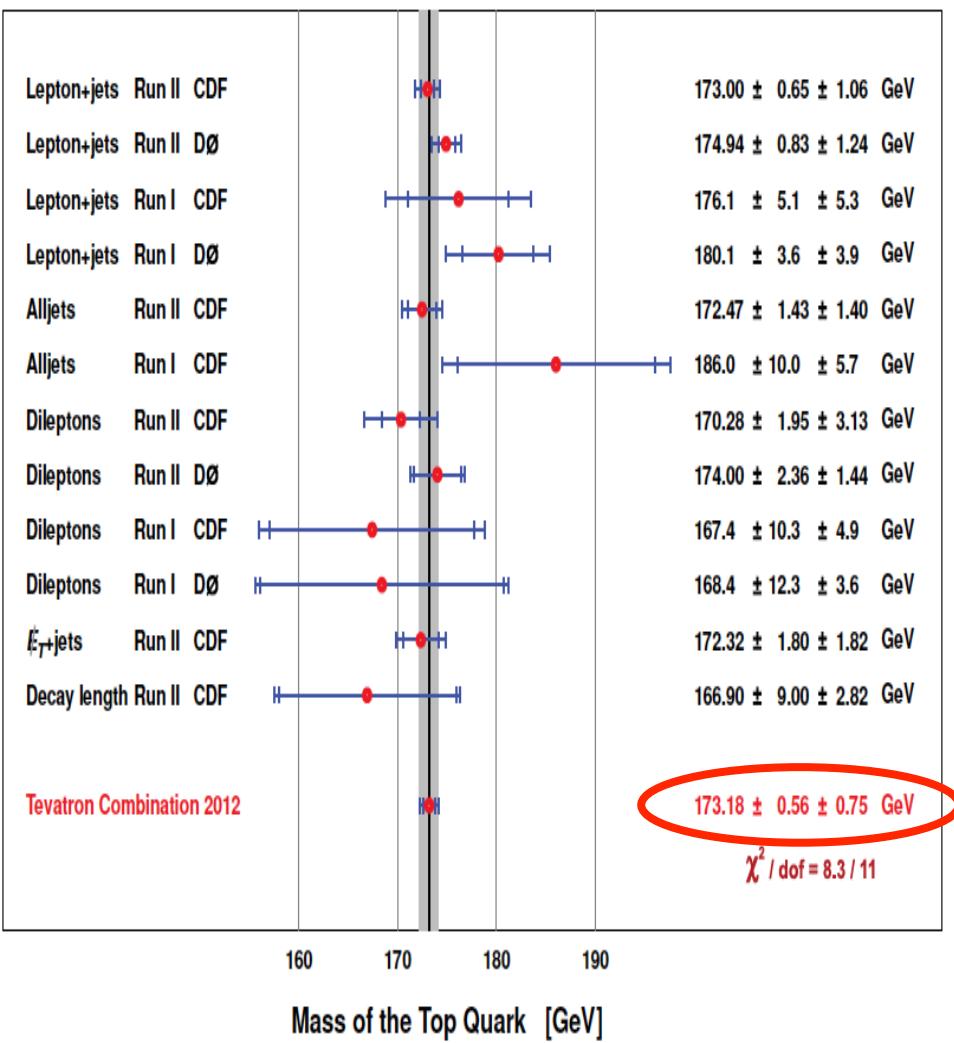


Mass of the W Boson



March 2012

Tevatron result from: arXiv:1207.1069



Used here:  $m_t = 173.18 \pm 0.94$  GeV  
 Moriond'13:  $m_t = 173.20 \pm 0.87$  GeV

# The electromagnetic coupling

- The EW fit requires precise knowledge of  $\alpha(M_Z)$  – better than 1% level
  - Enters various places: hadr. radiator functions, predictions of  $M_W$  and  $\sin^2\theta_{\text{eff}}^f$
- Conventionally parametrized as ( $\alpha(0) = \text{fine structure constant}$ ) :

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

- Evolution with renormalization scale:

$$\Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s)$$

- Leptonic term known up to three loops (for  $q^2 \gg m_l$ )
- Top quark contribution known up to 2 loops, *small*:  $-0.7 \times 10^{-4}$

[M. Steinhauser,  
PLB 429, 158 (1998)]

- Hadronic contribution (from the 5 light quarks) is difficult to calculate, cannot be obtained from pQCD alone.

- Analysis of low-energy  $e^+e^-$  data
- Usage of pQCD if lack of data

}

$$\boxed{\Delta\alpha_{\text{had}}^{(5)}(M_Z) = (274.9 \pm 1.0) \cdot 10^{-4}}$$

- Similar analysis to evaluation of hadronic contribution to  $(g-2)_\mu$

[M. Davier et al., Eur. Phys. J. C71, 1515 (2011)]

# Theoretical input

- In EW fit with Gfitter we use state-of-the-art calculations:
  - $M_W$  Mass of the W boson [M. Awramik et al., Phys. Rev. D69, 053006 (2004)]
  - $\sin^2\theta_{\text{eff}}$  Effective weak mixing angle [M. Awramik et al., JHEP 11, 048 (2006), M. Awramik et al., Nucl.Phys.B813:174-187 (2009)]
    - Full two-loop + leading beyond-two-loop form factor corrections
  - $\Gamma_{\text{had}}$  QCD Adler functions at  $N^3\text{LO}$  [P. A. Baikov et al., PRL108, 222003 (2012)]
    - $N^3\text{LO}$  prediction of the hadronic cross section
  - $(R_b)$  Partial width of  $Z \rightarrow b\bar{b}$  [Freitas et al., JHEP08, 050 (2012)] ← New!  
full 2-loop calc.
- Two nuisance parameters in EW fit for theoretical uncertainties:
  - $\delta M_W$  (4 MeV),  $\delta \sin^2\theta'_{\text{eff}}$  ( $4.7 \times 10^{-5}$ )
- Radiative corrections are important!
  - E.g. consider tree-level EW unification relation: 
$$M_W^2 \Big|_{\text{tree-level}} = \frac{M_Z^2}{2} \cdot \left( 1 + \sqrt{1 - \frac{\sqrt{8\pi\alpha}}{G_F M_Z^2}} \right)$$
    - This predicts:  $M_W = (79.964 \pm 0.005) \text{ GeV}$
    - Experiment:  $M_W = (80.385 \pm 0.015) \text{ GeV}$
- Without loop corrections:  $27\sigma$  discrepancy!

# Electroweak fit – Experimental inputs



## ■ Latest experimental inputs:

- **Z-pole observables:** from LEP / SLC  
[ADLO+SLD, Phys. Rept. 427, 257 (2006)]
- **$M_W$  and  $\Gamma_W$**  from LEP/Tevatron  
[arXiv:1204.0042]
- **$m_{\text{top}}$** : average from Tevatron  
[arXiv:1207.1069]
- **$m_c$ ,  $m_b$**  world averages (PDG)  
[PDG, J. Phys. G33,1 (2006)]
- **$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$  including  $\alpha_S$  dependency**  
[Davier et al., EPJC 71, 1515 (2011)]
- **$M_H$  from LHC**  
[arXiv:1207.7214, arXiv:1207.7235]

## ■ 7+2 free fit parameters:

- $M_Z$ ,  $M_H$ ,  $\alpha_S(M_Z^2)$ ,  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ ,  
 $m_t$ ,  $\bar{m}_c$ ,  $\bar{m}_b$
- 2 theory nuisance parameters
  - $\delta M_W$  (4 MeV),  $\delta \sin^2\theta'_{\text{eff}}$  ( $4.7 \times 10^{-5}$ )

$M_H$ [GeV] <sup>(<math>\circ</math>)</sup>	$125.7 \pm 0.4$	LHC
$M_W$ [GeV]	$80.385 \pm 0.015$	Tevatron
$\Gamma_W$ [GeV]	$2.085 \pm 0.042$	
$M_Z$ [GeV]	$91.1875 \pm 0.0021$	LHC
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	
$\sigma_{\text{had}}^0$ [nb]	$41.540 \pm 0.037$	SLC
$R_\ell^0$	$20.767 \pm 0.025$	
$A_{\text{FB}}^{0,\ell}$	$0.0171 \pm 0.0010$	SLC
$A_\ell^{(*)}$	$0.1499 \pm 0.0018$	
$\sin^2\theta_{\text{eff}}^\ell(Q_{\text{FB}})$	$0.2324 \pm 0.0012$	LEP
$A_c$	$0.670 \pm 0.027$	
$A_b$	$0.923 \pm 0.020$	LEP
$A_{\text{FB}}^{0,c}$	$0.0707 \pm 0.0035$	
$A_{\text{FB}}^{0,b}$	$0.0992 \pm 0.0016$	Tevatron
$R_c^0$	$0.1721 \pm 0.0030$	
$R_b^0$	$0.21629 \pm 0.00066$	
$\bar{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\bar{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
$m_t$ [GeV]	$173.18 \pm 0.94$	
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ $(\triangle\triangledown)$	$2757 \pm 10$	

# Electroweak Fit – SM Fit Results



- From the Gfitter Group, EPJC 72, 2205 (2012)

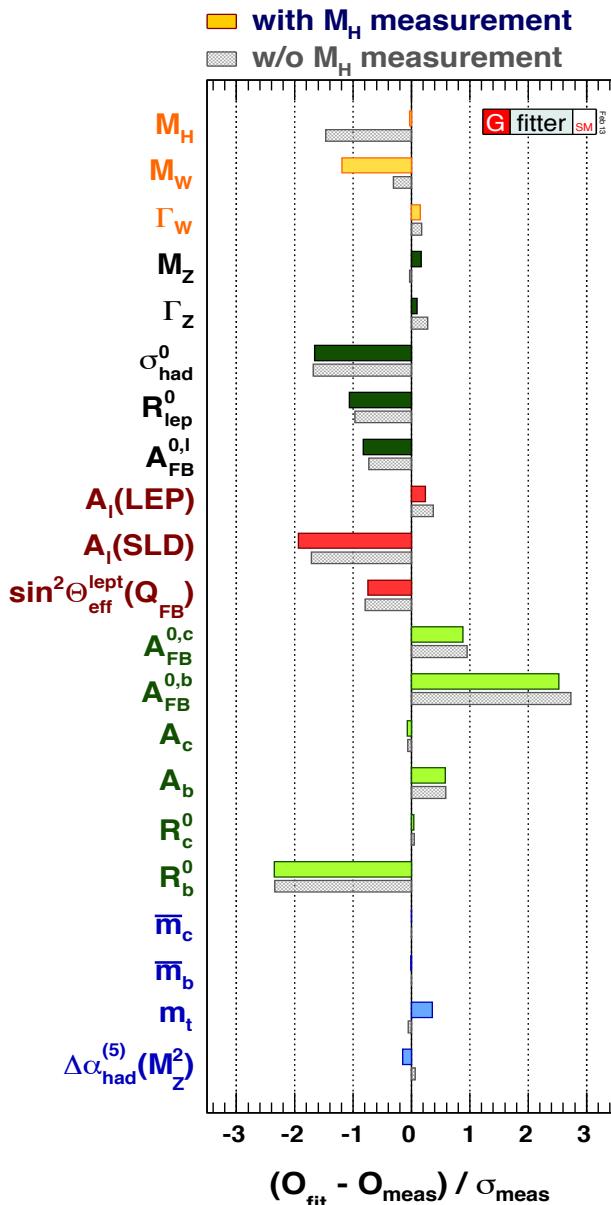
- Left: full fit incl.  $M_H$

- Middle: fit not incl.  $M_H$

- Right: fit incl  $M_H$ , not the row

Parameter	Input value	Free in fit	Fit result incl. $M_H$	Fit result not incl. $M_H$	Fit result incl. $M_H$ but not exp. input in row
$M_H$ [GeV] <sup>(o)</sup>	$125.7 \pm 0.4$	yes	$125.7 \pm 0.4$	$94^{+25}_{-22}$	$94^{+25}_{-22}$
$M_W$ [GeV]	$80.385 \pm 0.015$	–	$80.367 \pm 0.007$	$80.380 \pm 0.012$	$80.359 \pm 0.011$
$\Gamma_W$ [GeV]	$2.085 \pm 0.042$	–	$2.091 \pm 0.001$	$2.092 \pm 0.001$	$2.091 \pm 0.001$
$M_Z$ [GeV]	$91.1875 \pm 0.0021$	yes	$91.1878 \pm 0.0021$	$91.1874 \pm 0.0021$	$91.1983 \pm 0.0116$
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	–	$2.4954 \pm 0.0014$	$2.4958 \pm 0.0015$	$2.4951 \pm 0.0017$
$\sigma_{\text{had}}^0$ [nb]	$41.540 \pm 0.037$	–	$41.479 \pm 0.014$	$41.478 \pm 0.014$	$41.470 \pm 0.015$
$R_\ell^0$	$20.767 \pm 0.025$	–	$20.740 \pm 0.017$	$20.743 \pm 0.018$	$20.716 \pm 0.026$
$A_{FB}^{0,\ell}$	$0.0171 \pm 0.0010$	–	$0.01627 \pm 0.0002$	$0.01637 \pm 0.0002$	$0.01624 \pm 0.0002$
$A_\ell$ <sup>(*)</sup>	$0.1499 \pm 0.0018$	–	$0.1473^{+0.0006}_{-0.0008}$	$0.1477 \pm 0.0009$	$0.1468 \pm 0.0005^{(\dagger)}$
$\sin^2\theta_{\text{eff}}^\ell(Q_{\text{FB}})$	$0.2324 \pm 0.0012$	–	$0.23148^{+0.00011}_{-0.00007}$	$0.23143^{+0.00010}_{-0.00012}$	$0.23150 \pm 0.00009$
$A_c$	$0.670 \pm 0.027$	–	$0.6680^{+0.00025}_{-0.00038}$	$0.6682^{+0.00042}_{-0.00035}$	$0.6680 \pm 0.00031$
$A_b$	$0.923 \pm 0.020$	–	$0.93464^{+0.00004}_{-0.00007}$	$0.93468 \pm 0.00008$	$0.93463 \pm 0.00006$
$A_{FB}^{0,c}$	$0.0707 \pm 0.0035$	–	$0.0739^{+0.0003}_{-0.0005}$	$0.0740 \pm 0.0005$	$0.0738 \pm 0.0004$
$A_{FB}^{0,b}$	$0.0992 \pm 0.0016$	–	$0.1032^{+0.0004}_{-0.0006}$	$0.1036 \pm 0.0007$	$0.1034 \pm 0.0004$
$R_c^0$	$0.1721 \pm 0.0030$	–	$0.17223 \pm 0.00006$	$0.17223 \pm 0.00006$	$0.17223 \pm 0.00006$
$R_b^0$	$0.21629 \pm 0.00066$	–	$0.21474 \pm 0.00003$	$0.21475 \pm 0.00003$	$0.21473 \pm 0.00003$
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	$1.27^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	$4.20^{+0.17}_{-0.07}$	–
$m_t$ [GeV]	$173.18 \pm 0.94$	yes	$173.52 \pm 0.88$	$173.14 \pm 0.93$	$175.8^{+2.7}_{-2.4}$
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ $(\triangle\triangledown)$	$2757 \pm 10$	yes	$2755 \pm 11$	$2757 \pm 11$	$2716^{+49}_{-43}$
$\alpha_S(M_Z^2)$	–	yes	$0.1191 \pm 0.0028$	$0.1192 \pm 0.0028$	$0.1191 \pm 0.0028$
$\delta_{\text{th}} M_W$ [MeV]	$[-4, 4]_{\text{theo}}$	yes	4	4	–
$\delta_{\text{th}} \sin^2\theta_{\text{eff}}^\ell$ $(\triangle)$	$[-4.7, 4.7]_{\text{theo}}$	yes	-1.4	4.7	–

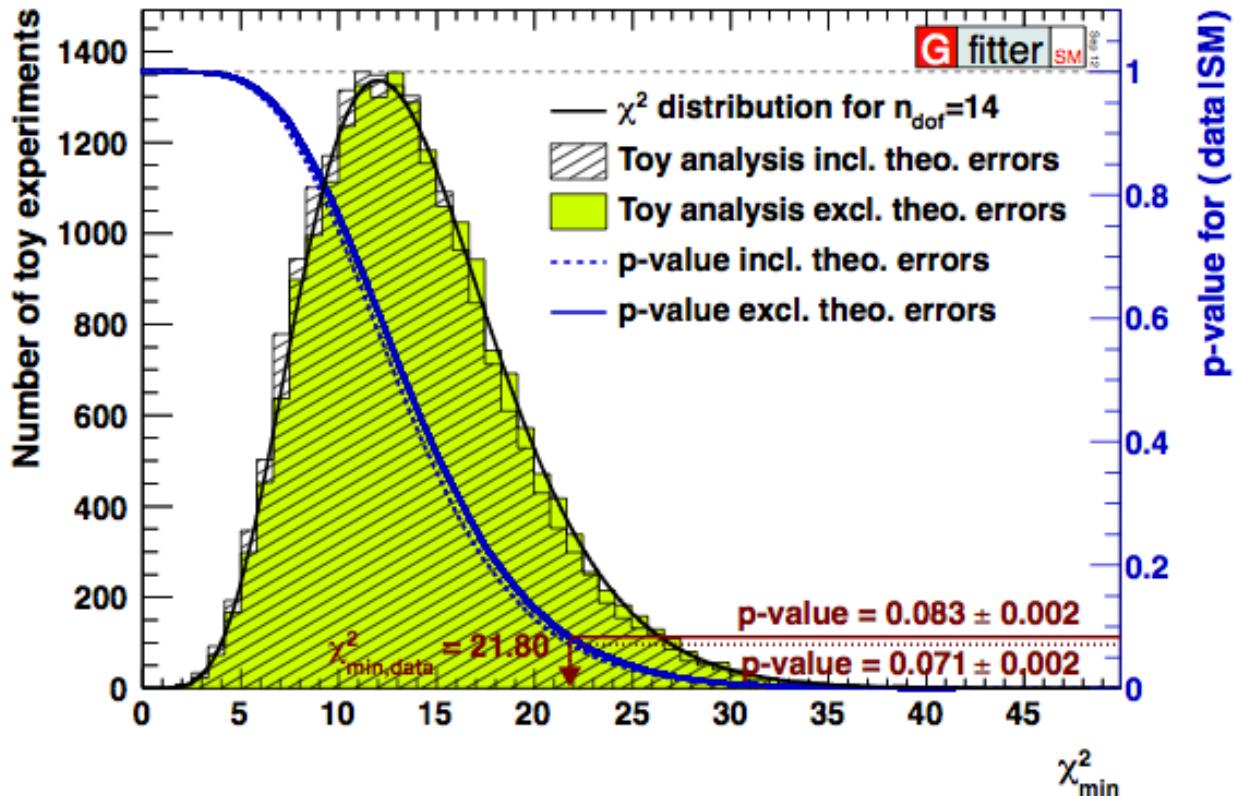
# Electroweak Fit – SM Fit Results



Plot inspired by Eberhardt et al. [arXiv:1209.1101]

- No individual value exceeds  $3\sigma$
- Small pulls for  $M_H$ ,  $M_Z$ ,  $\Delta\alpha_{had}^{(5)}(M_Z^2)$ ,  $\bar{m}_c$ ,  $\bar{m}_b$  indicate that input accuracies exceed fit requirements
- Largest deviations in b-sector:  $A_{FB}^{0,b}$  and  $R^0_b$  with  $2.5\sigma$  and  $-2.4\sigma$ 
  - → largest contribution to  $\chi^2$
- $R^0_b$  using one-loop calculation  $-0.8\sigma$ 
  - $R^0_b$  has only little dependence on  $M_H$
- Most affected when including  $M_H$  :  $M_W$  prediction:
  - Shift in predicted  $M_W$  value of 13 MeV.

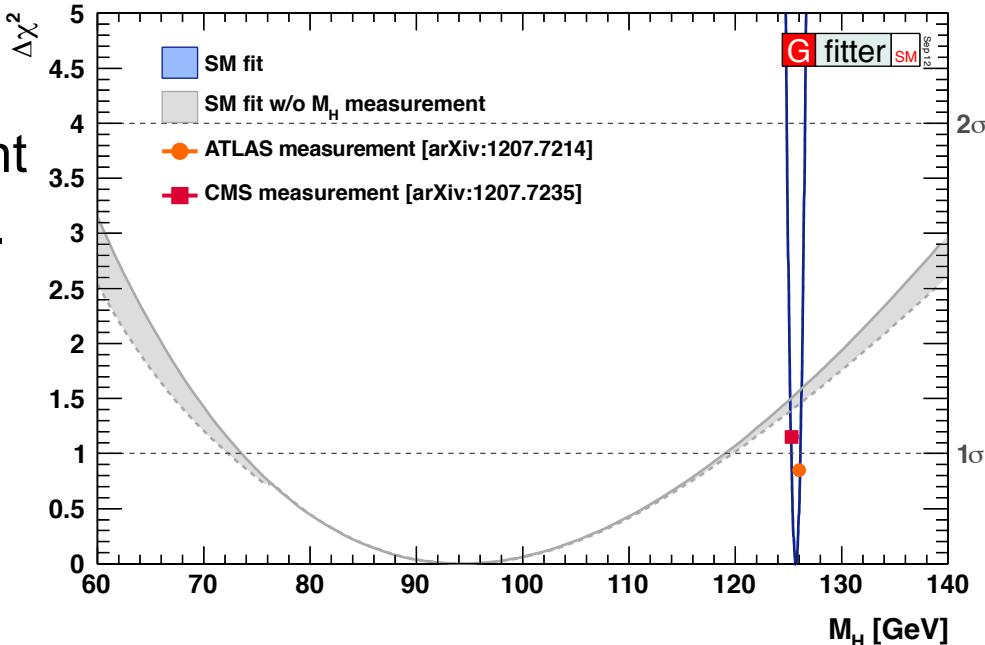
# Goodness of Fit



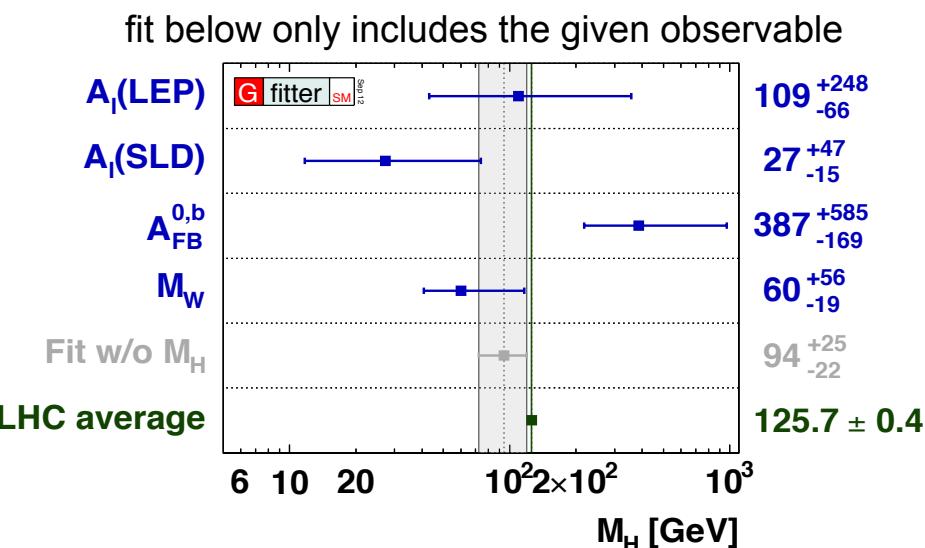
- Toy analysis: p-value for wrongly rejecting the SM =  $0.07^{+0.01}$  (theo)
  - p-value is equivalent to  $1.8\sigma$ .
  - Evaluated with 20k pseudo experiments – follows  $\chi^2$  with 14 d.o.f.
  - For comparison:  $\chi^2_{\text{min}} = 21.8 \rightarrow \text{Prob}(\chi^2_{\text{min}}, 14) = 8\%$
- Large value of  $\chi^2_{\text{min}}$  not due to inclusion of  $M_H$  measurement.
  - Without  $M_H$  measurement:  $\chi^2_{\text{min}} = 20.3 \rightarrow \text{Prob}(\chi^2_{\text{min}}, 13) = 9\%$

# Higgs results of the EW fit

- Scan of  $\Delta\chi^2$  profile versus  $M_H$ 
  - Grey band: fit w/o  $M_H$  measurement
  - Blue line: full SM fit, with  $M_H$  meas.
  - Fit w/o  $M_H$  measurement gives:  
 $M_H = 94^{+25}_{-22}$  GeV
  - Consistent at  $1.3\sigma$  with LHC measurement.



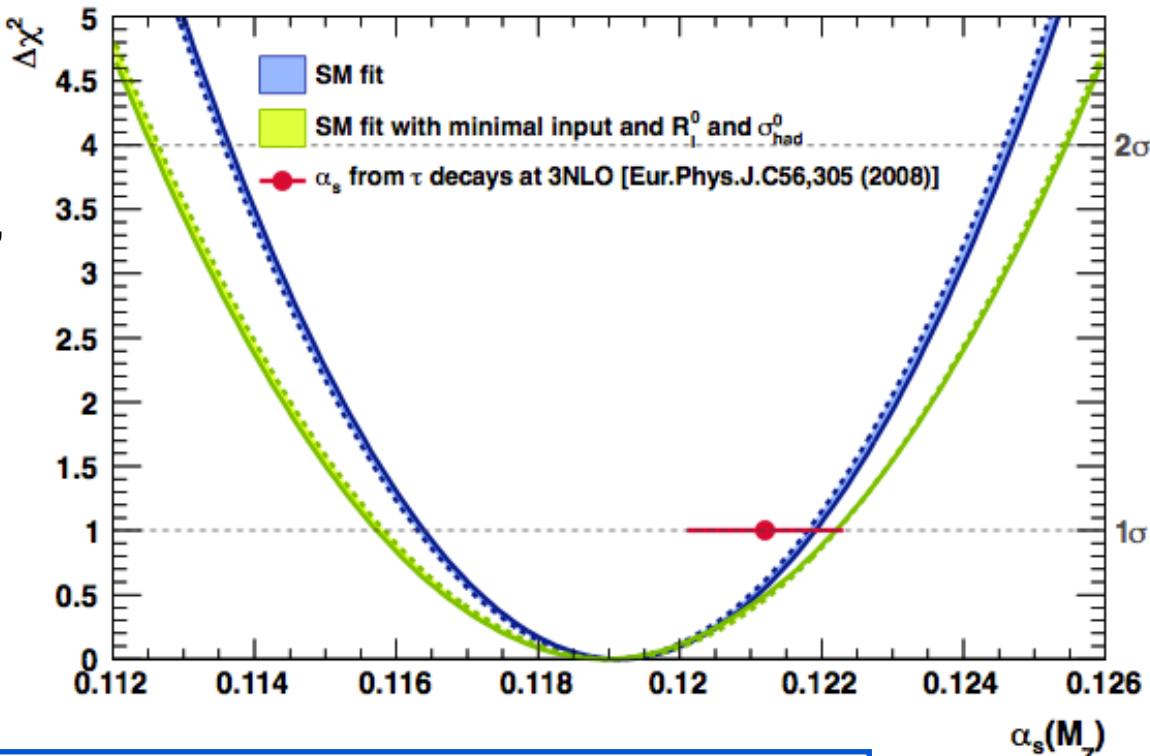
- Bottom plot: impact of other most sensitive Higgs observables
  - Determination of  $M_H$  removing all sensitive observables except the given one.
  - Known tension ( $2.5\sigma$ ) between  $A_l(\text{SLD})$ ,  $A_{FB}^{0,b}$ , and  $M_W$  clearly visible.



# Prediction for $\alpha_s(M_Z)$ from $Z \rightarrow$ hadrons



- Scan of  $\Delta\chi^2$  versus  $\alpha_s$
- Also shown: SM fit with minimal inputs:  
 $M_Z$ ,  $G_F$ ,  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ ,  $\alpha_s(M_Z)$ ,  
 $M_H$ , and fermion masses
- Determination of  $\alpha_s$  at  $N^3\text{LO}$ .
- Most sensitive through total hadronic cross-section  $\sigma_{\text{had}}^0$  and partial leptonic width  $R_L^0$

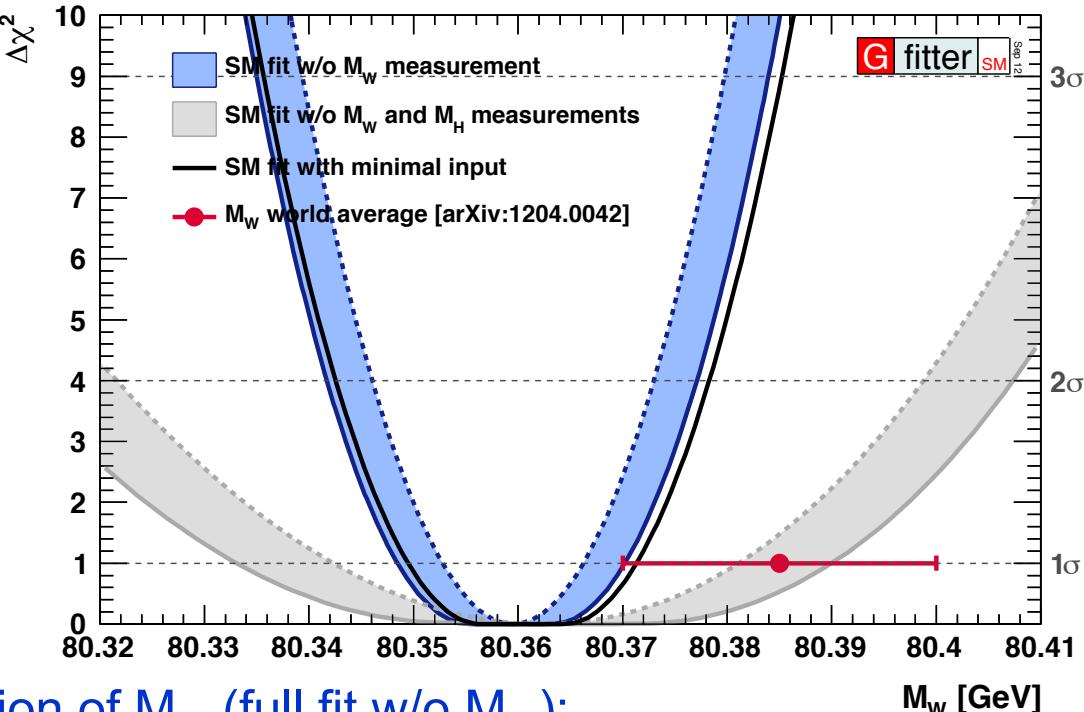


$$\alpha_s(M_Z) = 0.1191 \pm 0.0028 \text{ (exp.)} \pm 0.0001 \text{ (theo.)}$$

- Theory uncertainty at per-mille level (obtained by scale variation of  $\Gamma_{\text{had}}$ ).
- In good agreement with value from  $\tau$  decays, also at  $N^3\text{LO}$ .*
- (Improvements in precision only expected with ILC/GigaZ. See later.)

# Indirect determination of W mass

- Scan of  $\Delta\chi^2$  profile versus  $M_W$ 
  - Also shown: SM fit with minimal inputs:  $M_Z$ ,  $G_F$ ,  $\Delta\alpha_{had}^{(5)}(M_Z)$ ,  $\alpha_s(M_Z)$ ,  $M_H$ , and fermion masses
  - Good consistency between total fit and SM w/ minimal inputs
- $M_H$  measurement allows for precise constraint on  $M_W$ 
  - Agreement at  $1.4\sigma$
- Fit result for indirect determination of  $M_W$  (full fit w/o  $M_W$ ):



$$\begin{aligned}
 M_W &= 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{had}} \\
 &\quad \pm 0.0017_{\alpha_s} \pm 0.0002_{M_H} \pm 0.0040_{\text{theo}} \\
 &= 80.359 \pm 0.011_{\text{tot}}
 \end{aligned}$$

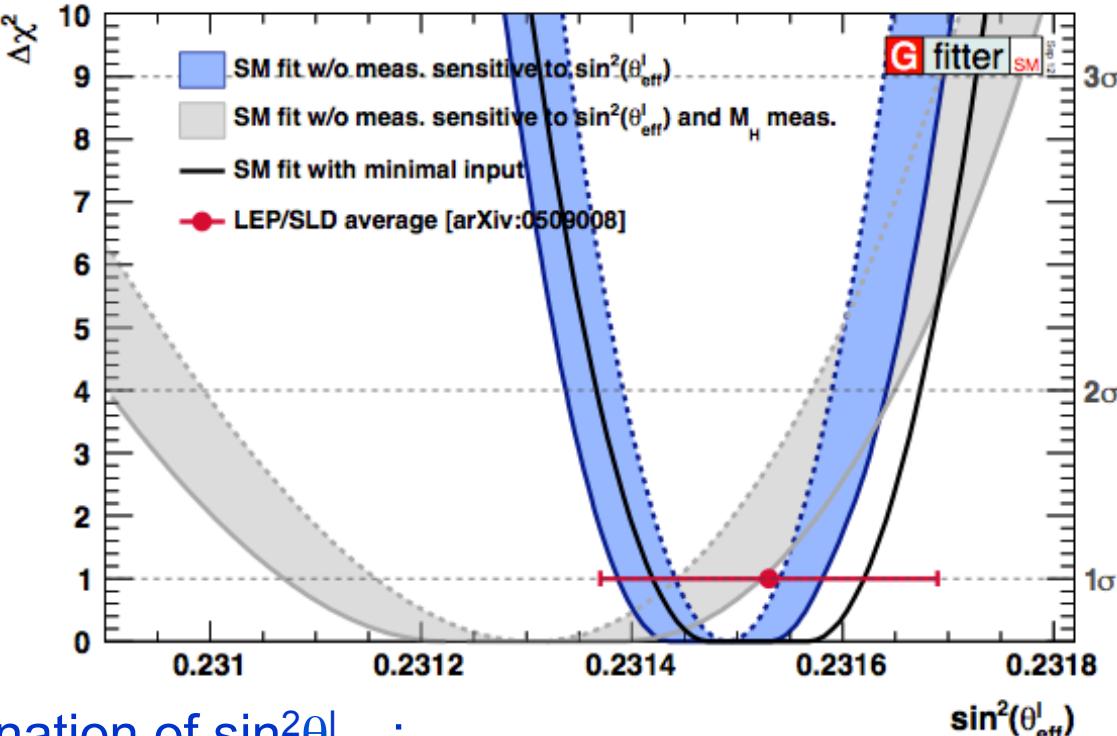
- More precise estimate of  $M_W$  than the direct measurements!
  - Uncertainty on world average measurement: 15 MeV

# Indirect effective weak mixing angle

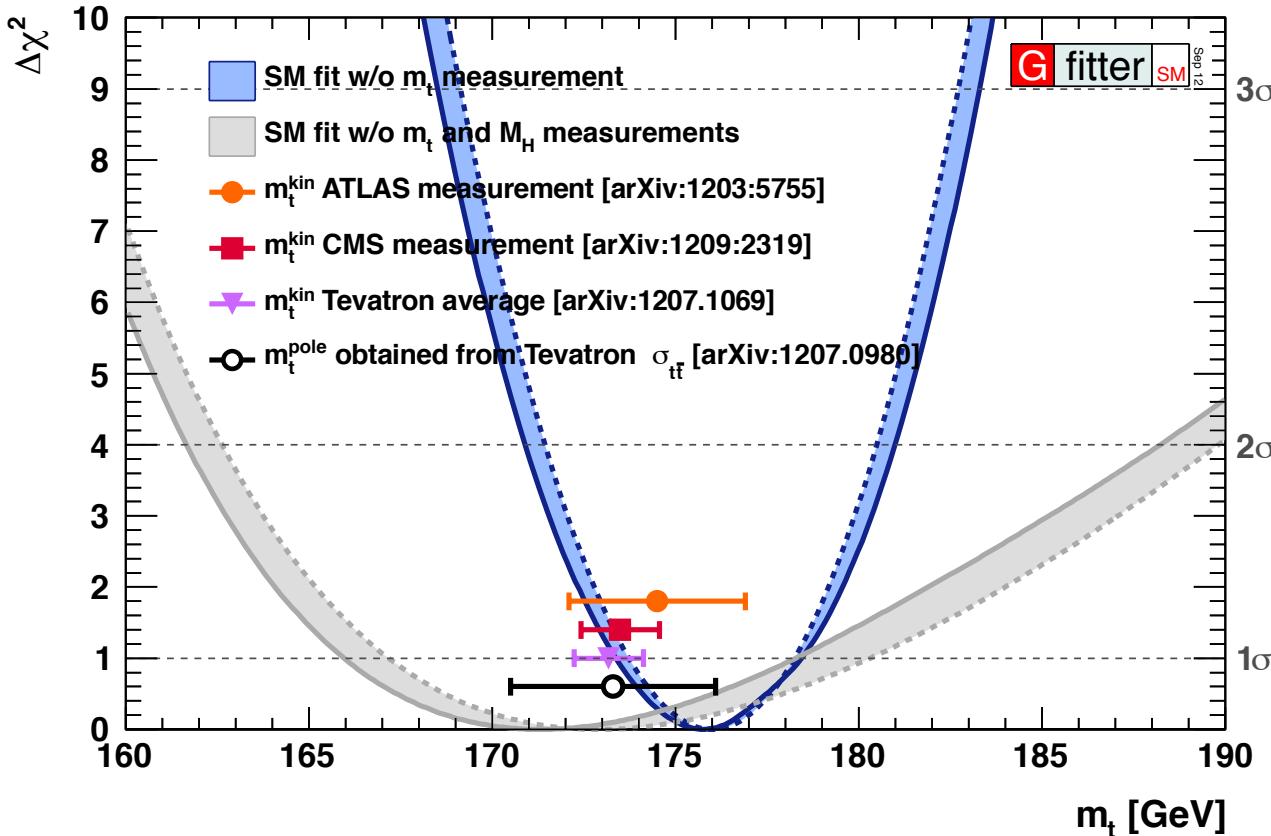
- Right: scan of  $\Delta\chi^2$  profile versus  $\sin^2\theta_{\text{eff}}^l$ 
  - All sensitive measurements removed from the SM fit.
  - Also shown: SM fit with minimal inputs
- $M_H$  measurement allows for very precise constraint on  $\sin^2\theta_{\text{eff}}^l$
- Fit result for indirect determination of  $\sin^2\theta_{\text{eff}}^l$ :

$$\begin{aligned} \sin^2\theta_{\text{eff}}^l &= 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\text{had}}} \\ &\quad \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{\text{theo}}, \\ &= 0.23150 \pm 0.00010_{\text{tot}}, \end{aligned}$$

- More precise than direct determination (from LEP/SLD) !
  - Uncertainty on LEP/SLD average:  $1.6 \times 10^{-4}$



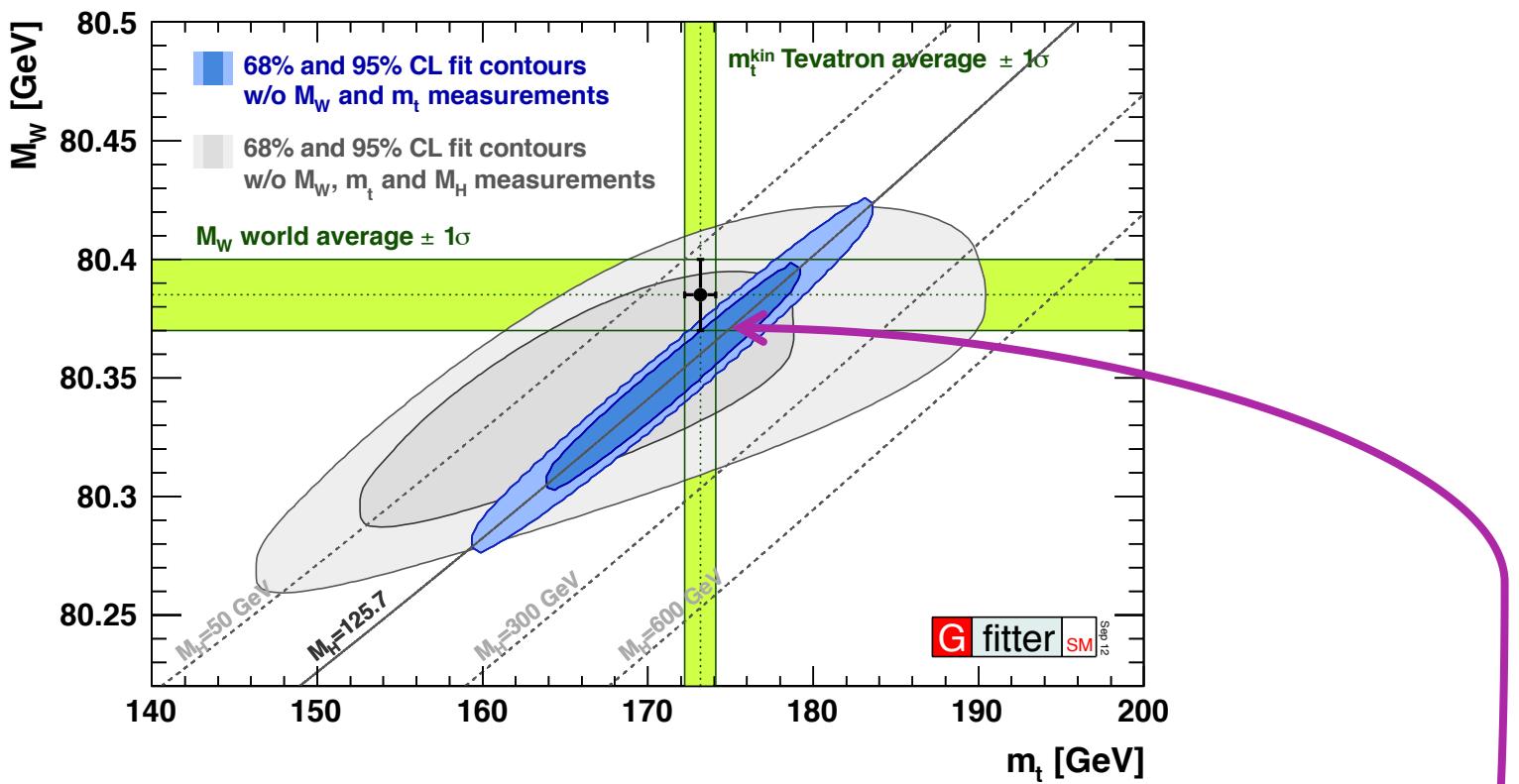
# Indirect determination of top mass



- Shown: scan of  $\Delta\chi^2$  profile versus  $m_t$  (without  $m_t$  measurement)
  - $M_H$  measurement allows for significant better constraint of  $m_t$
  - Indirect determination consistent with direct measurements
    - Remember: fully obtained from loop corrections!
- Indirect result:  $m_t = 175.8^{+2.7}_{-2.4}$  GeV (Tevatron w.a.:  $173.2 \pm 0.9$  GeV)

# State of the SM: W versus top mass

- Scan of  $M_W$  vs  $m_t$ , with the direct measurements excluded from the fit.
- Results from Higgs measurement significantly reduces allowed indirect parameter space → corners the SM!

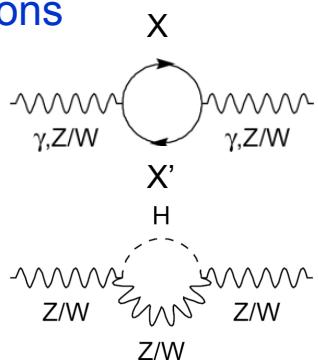


- Observed agreement demonstrates impressive consistency of the SM!

# Constraints on Oblique Corrections



- At low energies, BSM physics appears dominantly through vacuum polarization corrections
  - Aka, “oblique corrections”
- Oblique corrections reabsorbed into electroweak form factors
  - $\Delta\rho$ ,  $\Delta\kappa$ ,  $\Delta r$  parameters, appearing in:  $M_W^2$ ,  $\sin^2\theta_{\text{eff}}$ ,  $G_F$ ,  $\alpha$ , etc.
- Electroweak fit sensitive to BSM physics through oblique corrections
  - Similar to sensitivity to Higgs loop corrections.



- Oblique corrections from New Physics described through STU parametrization [Peskin and Takeuchi, Phys. Rev. D46, 1 (1991)]

$$O_{\text{meas}} = O_{\text{SM,REF}}(m_H, m_t) + c_S S + c_T T + c_U U$$

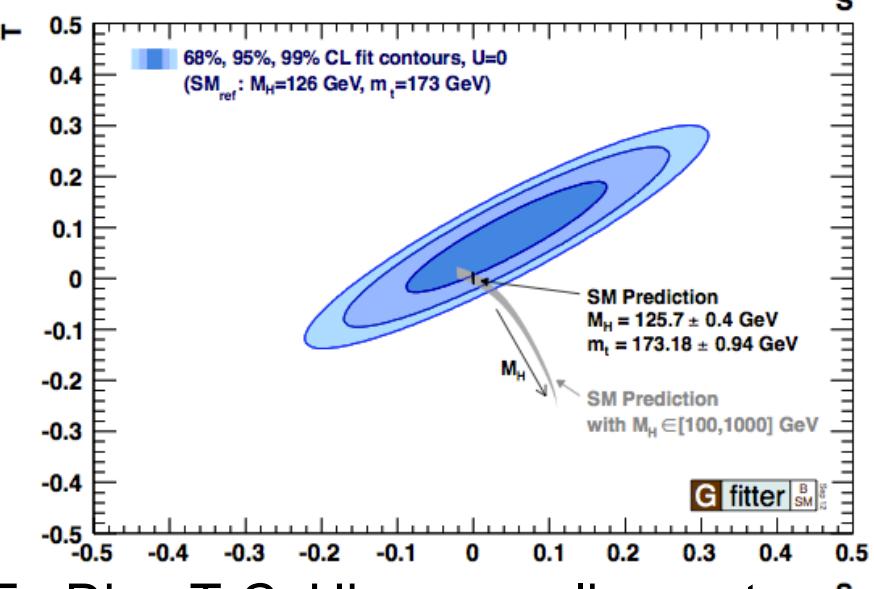
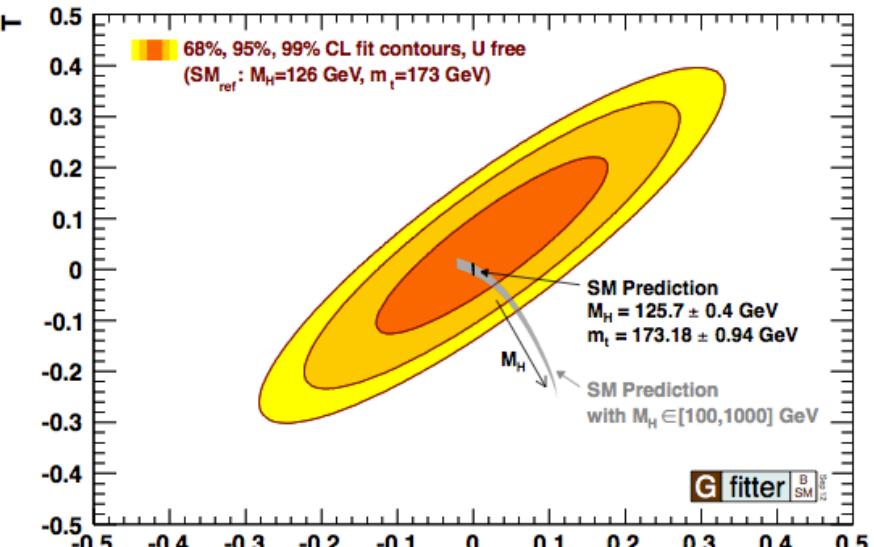
- **S** : New Physics contributions to neutral currents
- **T** : Difference between neutral and charged current processes – sensitive to weak isospin violation
- **U** : (+S) New Physics contributions to charged currents. U only sensitive to W mass and width, usually very small in NP models (often:  $U=0$ )
- Also implemented: correction to  $Z \rightarrow b\bar{b}$  coupling, extended parameters (VWX)

[Burgess et al., Phys. Lett. B326, 276 (1994)]  
 [Burgess et al., Phys. Rev. D49, 6115 (1994)]

# Fit results for S, T, U

- S,T,U obtained from fit to the EW observables
- SM:  $M_H = 126 \text{ GeV}$ ,  $m_t = 173 \text{ GeV}$ 
  - This defines  $(S, T, U) = (0, 0, 0)$
- SM: S, T depend logarithmically on  $M_H$
- Fit result:**

	S	T	U
$S = 0.03 \pm 0.10$	1	+0.89	-0.54
$T = 0.05 \pm 0.12$		1	-0.83
$U = 0.03 \pm 0.10$			1
- Stronger constraints from fit with  $U=0$ .
- Also available for  $Z \rightarrow b\bar{b}$  correction.
- No indication for new physics.
- Can now use this to constrain 4<sup>th</sup> gen, Ex-Dim, T-C, Higgs couplings, etc.





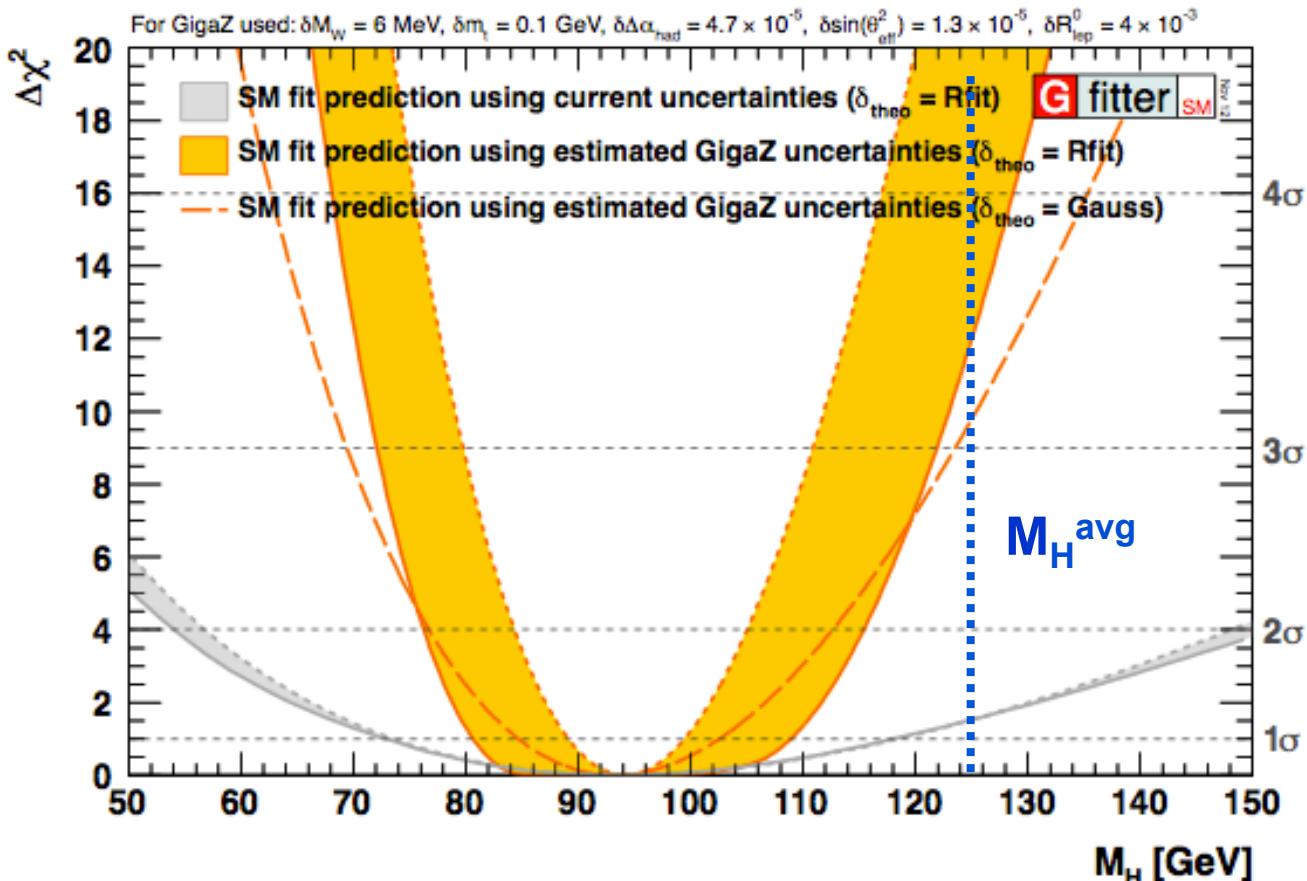
# ILC Prospects for the Standard Model fit

# Prospects for ILC with Giga Z



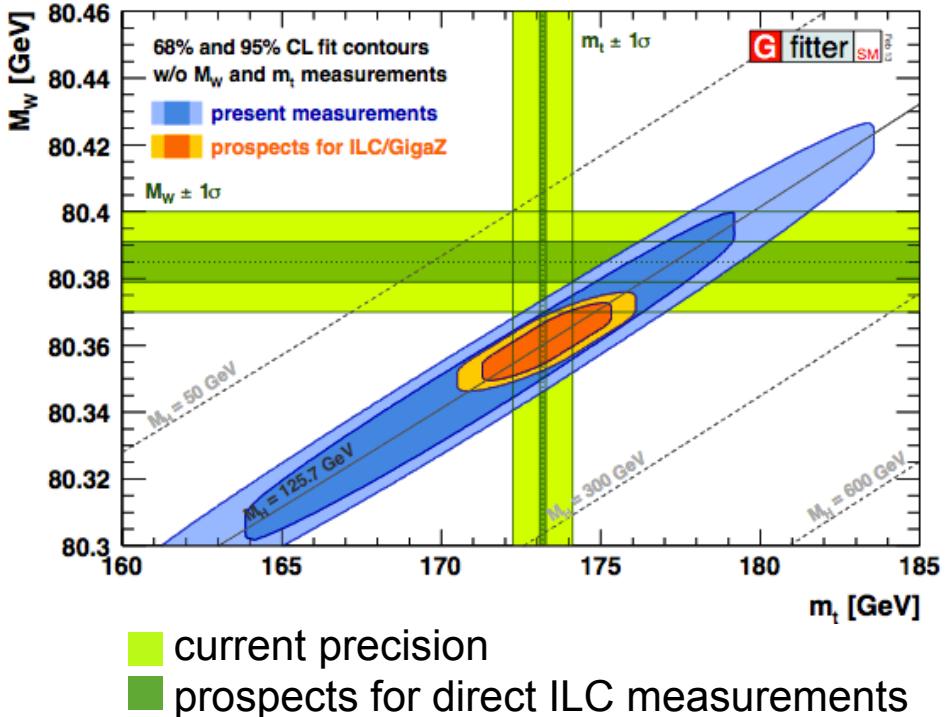
- Future Linear Collider could improve precision of EW observables tremendously.
  - *WW threshold, to obtain  $M_W$* 
    - from threshold scan:  $\delta M_W : 15 \rightarrow 6 \text{ MeV}$
  - *ttbar threshold, to obtain  $m_t$* 
    - obtain  $m_t$  indirectly from production cross section:  $\delta m_t : 0.9 \rightarrow 0.1 \text{ GeV}$
  - *Z pole measurements*
    - High statistics:  $10^9$  Z decays:  $\delta R^0_{\text{lep}} : 2.5 \cdot 10^{-2} \rightarrow 4 \cdot 10^{-3}$
    - With polarized beams, uncertainty on  $\delta A^{0,f}_{\text{LR}}$ :  $10^{-3} \rightarrow 10^{-4}$ , which translates to  $\delta \sin^2 \theta^l_{\text{eff}} : 1.6 \cdot 10^{-4} \rightarrow 1.3 \cdot 10^{-5}$
- Low-energy data results to improve  $\Delta \alpha_{\text{had}}$ :
  - ISR-based (BABAR) and KLOE-II, BESIII  $e^+e^-$  cross-section measurements, in particular around cc resonance ...
  - ... plus: improved  $\alpha_s$ , improvements in theory:  $\Delta \alpha_{\text{had}} : 10^{-4} \rightarrow 5 \cdot 10^{-5}$

# Prospects for ILC with Giga Z

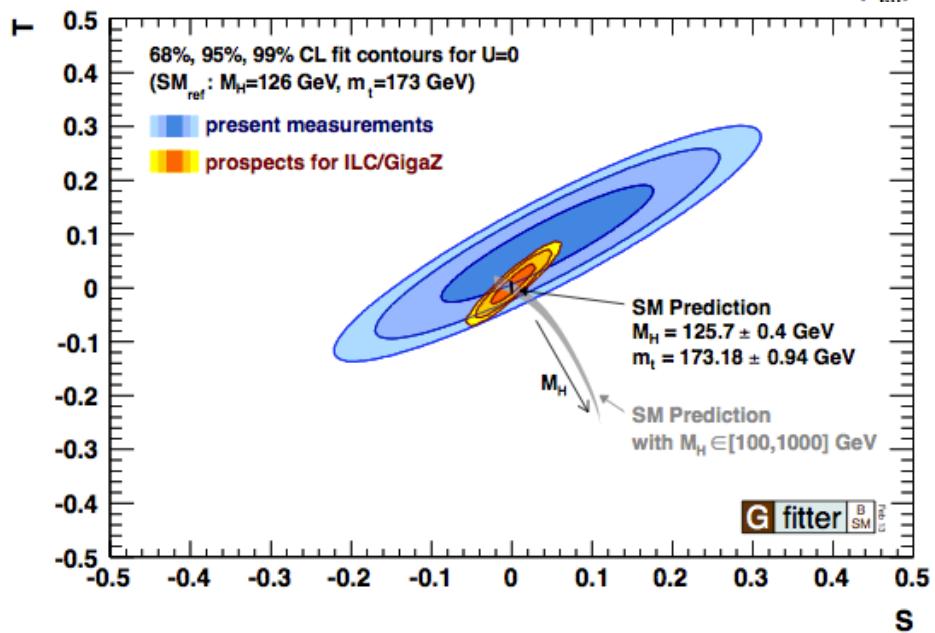
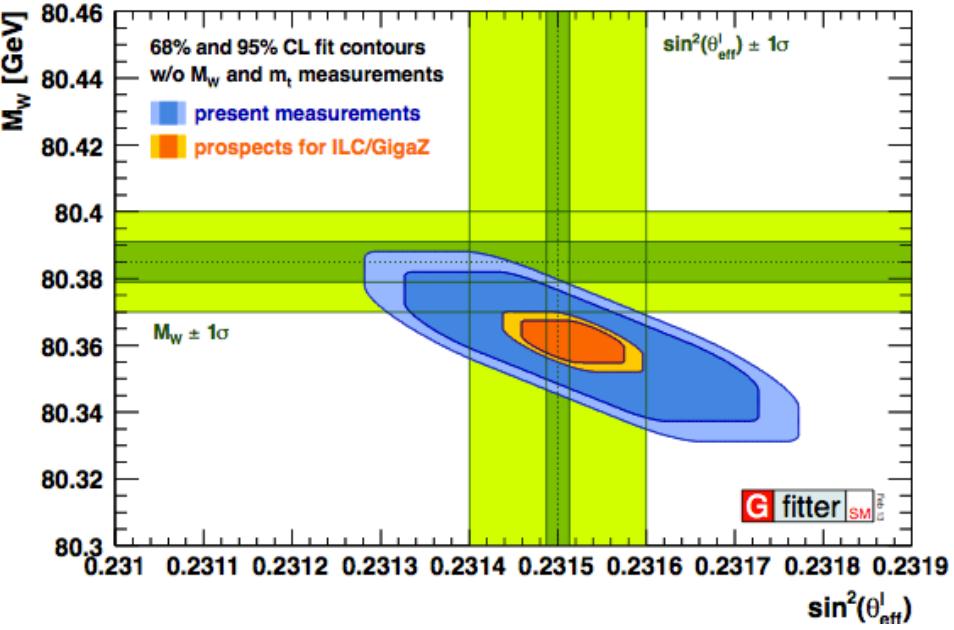


- Logarithmic dependency on  $M_H \rightarrow$  cannot compete with direct  $M_H$  meas.
- Indirect prediction  $M_H$  dominated by theory uncertainties.
  - No theory uncertainty:  $M_H = 94.2^{+5.3}_{-5.0} \text{ GeV}$
  - R-fit scheme:  $M_H = 92.3^{+16.6}_{-11.6} \text{ GeV}$

# Prospects for ILC with Giga Z



- Assuming also 50% of today's theoretical uncertainties
  - Implies three-loop EW calculations!
- Huge reduction of uncertainty on indirect determinations
- Also strong constraints on S, T, U



# Summary

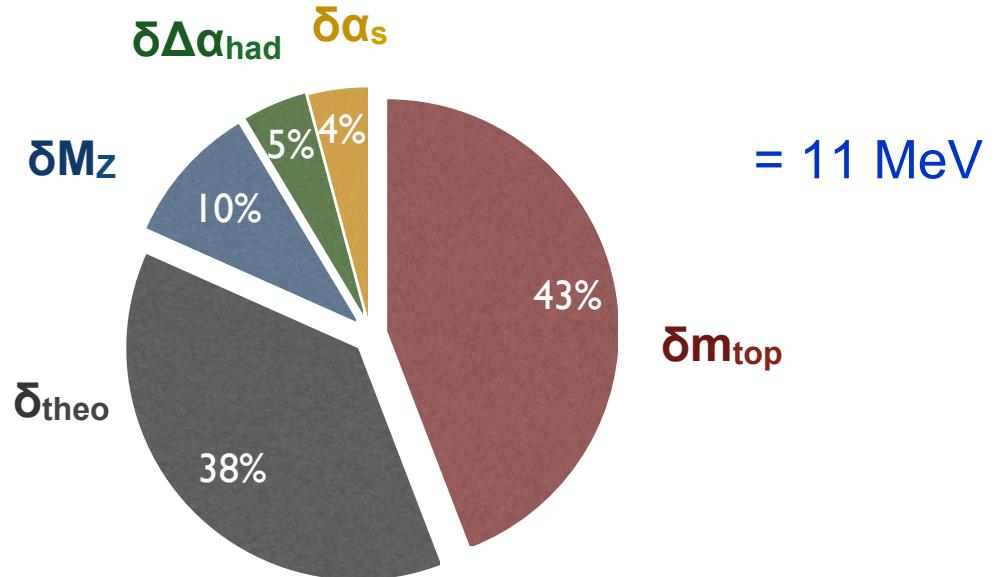


- Including  $M_H$  measurement, for first time SM is fully over-constrained!
  - $M_H$  consistent at  $1.3\sigma$  with indirect prediction from EW fit.
- p-Value of global electroweak fit of SM: 7% (pseudo-experiments)
  - Would be great to revisit  $Z \rightarrow b\bar{b}$ , both theoretically and experimentally
- Knowledge of  $M_H$  dramatically improves SM prediction of key observables
  - $M_W$  ( $28 \rightarrow 11$  MeV),  $\sin^2\theta_{\text{eff}}$  ( $2.3 \times 10^{-5} \rightarrow 1.0 \times 10^{-5}$ ),  $m_t$  ( $6.2 \rightarrow 2.5$  GeV)
- Improved accuracies set benchmark for new direct measurements!

# Outlook

- Paradigm shift for EW fit: from Higgs mass prediction to ... consistency tests of the Standard Model:

- $\delta M_W$  (indirect) =
  - Large contributions to  $\delta M_W$  (and  $\delta \sin^2 \theta_W^{eff}$ ) from top and unknown higher-order EW corrections.
- $\delta M_W$  (direct) = 15 MeV



- What's next for Gfitter:  
combine Higgs couplings in the EW fit. To be continued ...
- Latest results always available at: <http://cern.ch/Gfitter>
  - Results of this presentation: EPJC 72, 2205 (2012)



A **Generic Fitter** Project for HEP Model Testing

## Backup

# New $R^0_b$ calculation

[A. Freitas et al., JHEP 1208, 050 (2012)]



- The branching ratio  $R^0_b$ : partial decay width of  $Z \rightarrow b\bar{b}$  to  $Z \rightarrow q\bar{q}$
- Freitas et al: full 2-loop calculation of  $Z \rightarrow b\bar{b}$
- Contribution of same terms as in the calculation of  $\sin^2\theta_{\text{eff}}^{bb}$   
→ cross-check of two results found good agreement
- Two-loop corrections comparable to experimental uncertainty ( $6.6 \times 10^{-4}$ )

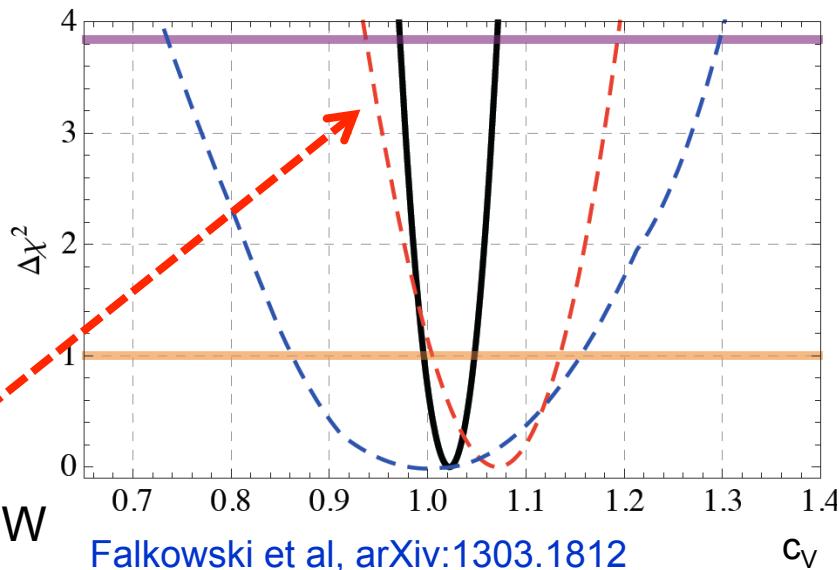
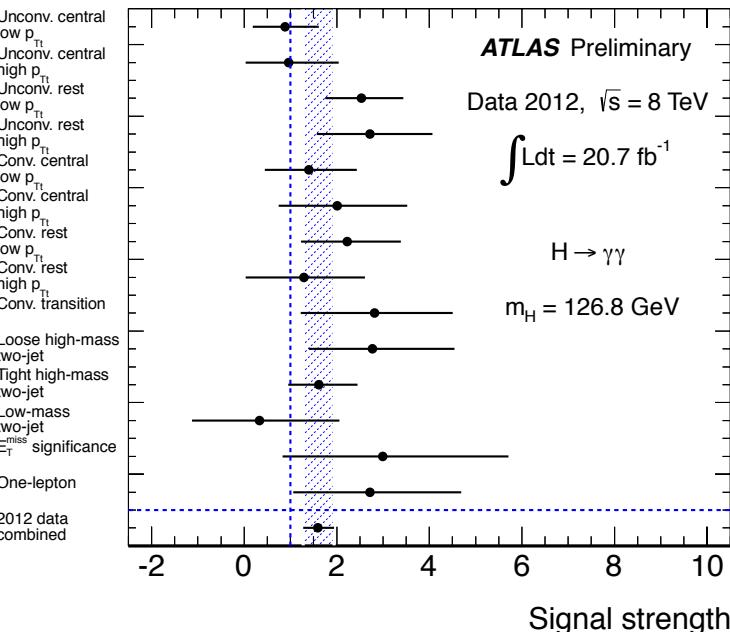
$M_H$ [GeV]	1-loop EW and QCD correction to FSR	2-loop EW correction	2-loop EW and 2+3-loop QCD correction to FSR	1+2-loop QCD correction to gauge boson self-energies
100	$\mathcal{O}(\alpha) + \text{FSR}_{1\text{-loop}}$ $[10^{-3}]$	$\mathcal{O}(\alpha_{\text{ferm}}^2)$ $[10^{-4}]$	$\mathcal{O}(\alpha_{\text{ferm}}^2) + \text{FSR}_{>1\text{-loop}}$ $[10^{-4}]$	$\mathcal{O}(\alpha\alpha_s, \alpha\alpha_s^2)$ $[10^{-4}]$
200	-3.632	-6.569	-9.333	-0.404
400	-3.651	-6.573	-9.332	-0.404
	-3.675	-6.581	-9.331	-0.404

# Higgs couplings in the EW fit

- In latest ATLAS  $H \rightarrow \gamma\gamma$ ,  $2.3\sigma$  deviation seen from SM  $\mu$  ( $\equiv 1.0$ )
- Interpret.:  $H \rightarrow VV$  couplings scaled with  $c_V$

From: Falkowski et al, arXiv:1303.1812

- Modified Higgs couplings can be constrained by EW fit through extended STU formalism.
- Result of  $c_V$  driven by limit on  $T$  parameter.
  - Tree-level relation:  $\rho_0 = \frac{M_{W_0}^2}{M_{Z_0}^2 c_W^2} = 1 + \alpha T$
  - $\alpha T \approx \frac{3g_Y^2}{32\pi^2} (c_V^2 - 1) \log(\Lambda/m_Z)$
  - Reminder:  $T = 0.05 \pm 0.12$  (Gfitter)
- EW-fit Falkowski et al:  $c_V \simeq 1.08 \pm 0.07$ 
  - Blue dashed:  $c_V$  from  $\mu$ 's, black: comb. w/ EW

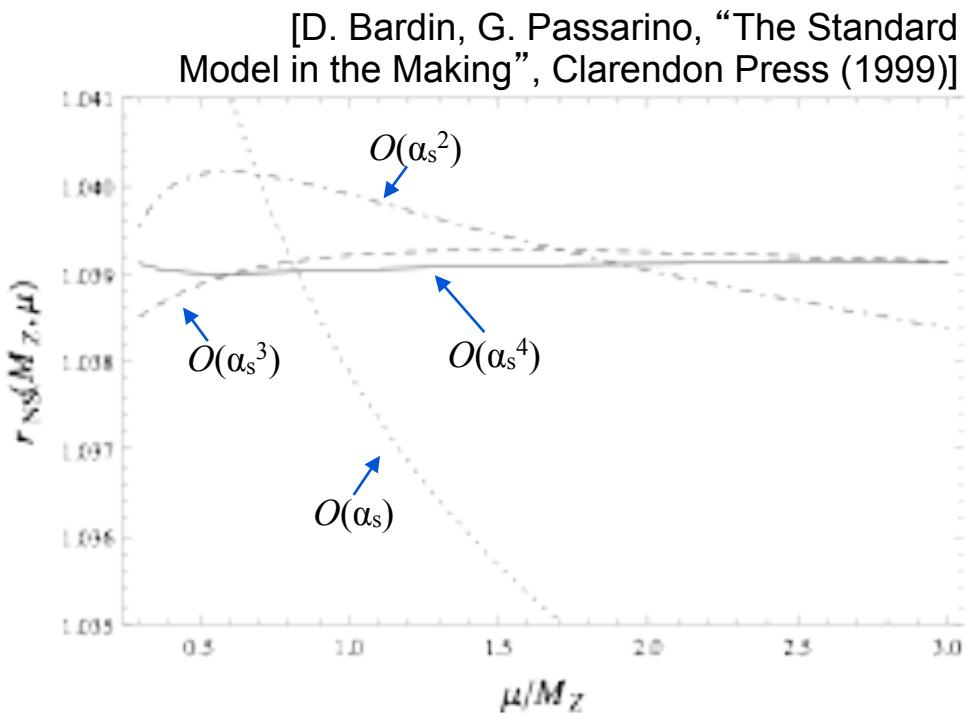


# Radiator Functions

- Partial widths are defined inclusively:  
contain both QCD and QED contributions.
- Corrections expressed as so-called radiator functions  $R_{A,f}$  and  $R_{V,f}$

$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left( |g_{A,f}|^2 R_{A,f} + |g_{V,f}|^2 R_{V,f} \right)^2$$

- High sensitivity to the strong coupling  $\alpha_s$
- Recently, full four-loop calculation of QCD Adler function became available ( $N^3LO$ )
- Much-reduced scale dependence!
- Theoretical uncertainty of 0.1 MeV, compared with experimental uncertainty of 2.0 MeV.

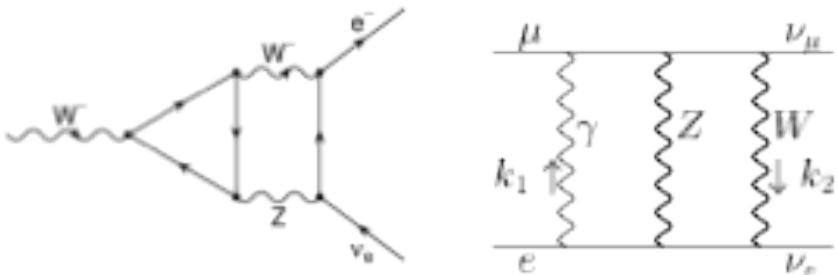


[P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)]  
 [P. Baikov et al., Phys. Rev. Lett. 104, 132004 (2010)]

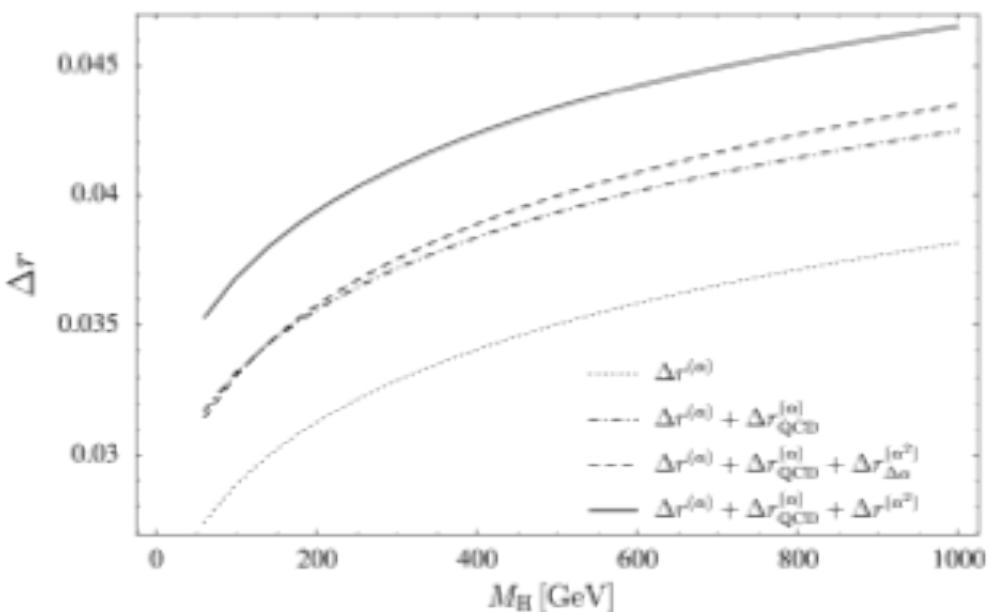
# Calculation of $M_W$

- Full EW one- and two-loop calculation of fermionic and bosonic contributions.
- One- and two-loop QCD corrections and leading terms of higher order corrections.
- Results for  $\Delta r$  include terms of order  $O(\alpha)$ ,  $O(\alpha\alpha_s)$ ,  $O(\alpha\alpha_s^2)$ ,  $O(\alpha_{\text{ferm}}^2)$ ,  $O(\alpha_{\text{bos}}^2)$ ,  $O(\alpha^2\alpha_s m_t^4)$ ,  $O(\alpha^3 m_t^6)$
- Uncertainty estimate:
  - Missing terms of order  $O(\alpha^2\alpha_s)$ : about 3 MeV (from  $O(\alpha^2\alpha_s m_t^4)$ )
  - Electroweak three-loop correction  $O(\alpha^3)$ : < 2 MeV
  - Three-loop QCD corrections  $O(\alpha\alpha_s^3)$ : < 2 MeV
- Total:  $\delta M_W \approx 4$  MeV

[M Awramik et al., Phys. Rev. D69, 053006 (2004)]  
 [M Awramik et al., Phys. Rev. Lett. 89, 241801 (2002)]



[A Freitas et al., Phys. Lett. B495, 338 (2000)]



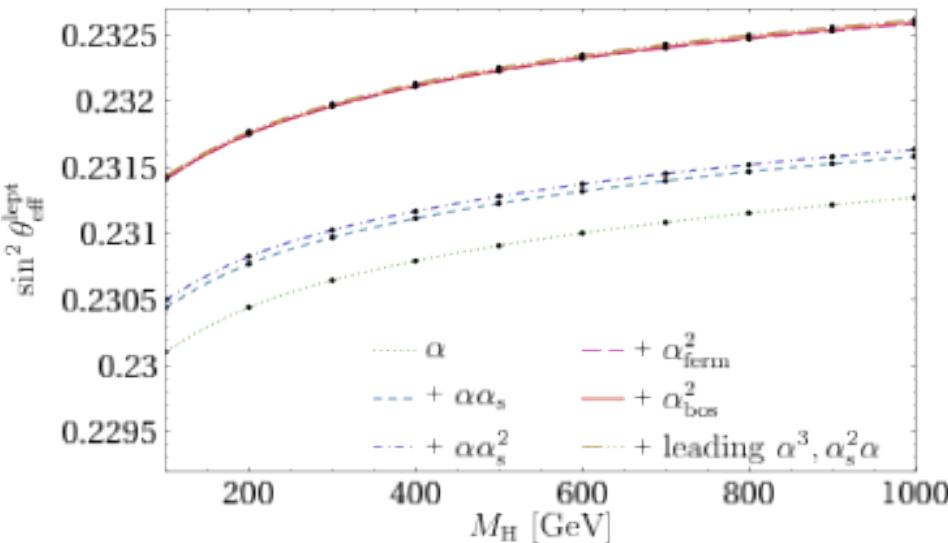
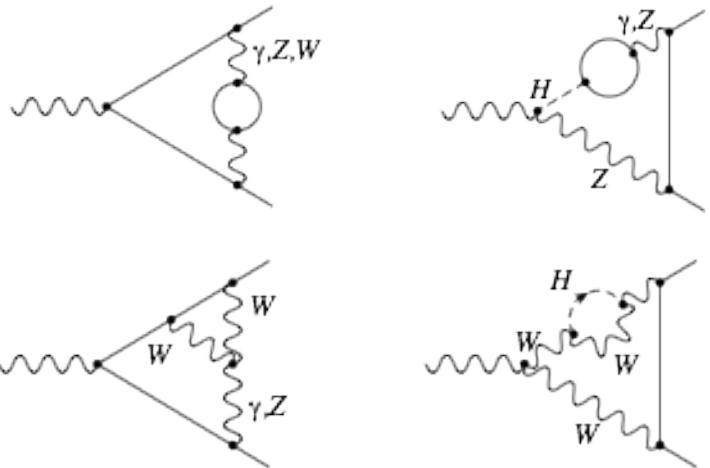
# Calculation of $\sin^2(\theta_{\text{eff}}^l)$

- Effective mixing angle:

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = (1 - M_W^2/M_Z^2) (1 + \Delta\kappa)$$

[M Awramik et al, Phys. Rev. Lett. 93, 201805 (2004)]  
 [M Awramik et al., JHEP 11, 048 (2006)]

- Two-loop EW and QCD correction to  $\Delta\kappa$  known, leading terms of higher order QCD corrections.
- Fermionic two-loop correction about  $10^{-3}$ , whereas bosonic one  $10^{-5}$ .
- Uncertainty estimate obtained with different methods, geometric progression, leading to total:  
 $\delta \sin^2(\theta_{\text{eff}}^l) = 4.7 \times 10^{-5}$



# Input correlations of the EW fit



- Input correlation coefficients between Z pole measurements

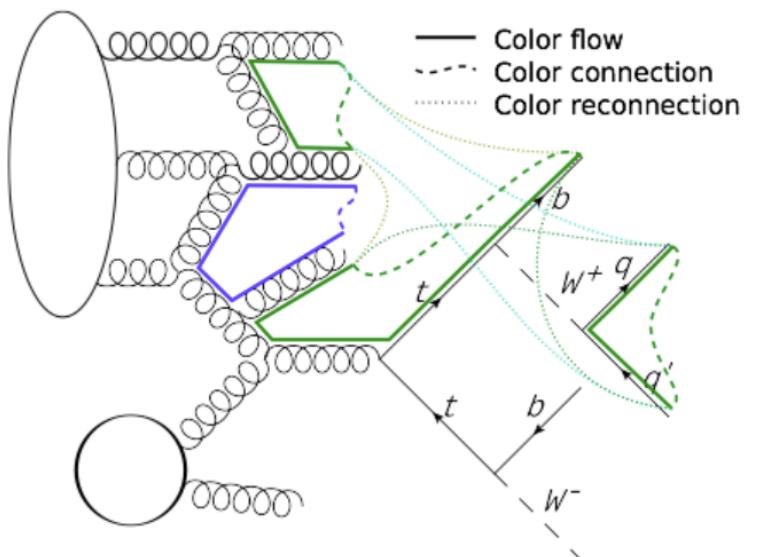
	$M_Z$	$\Gamma_Z$	$\sigma_{\text{had}}^0$	$R_\ell^0$	$A_{\text{FB}}^{0,\ell}$		$A_{\text{FB}}^{0,c}$	$A_{\text{FB}}^{0,b}$	$A_c$	$A_b$	$R_c^0$	$R_b^0$
$M_Z$	1	-0.02	-0.05	0.03	0.06	$A_{\text{FB}}^{0,c}$	1	0.15	0.04	-0.02	-0.06	0.07
$\Gamma_Z$		1	-0.30	0.00	0.00	$A_{\text{FB}}^{0,b}$		1	0.01	0.06	0.04	-0.10
$\sigma_{\text{had}}^0$			1	0.18	0.01	$A_c$			1	0.11	-0.06	0.04
$R_\ell^0$				1	-0.06	$A_b$				1	0.04	-0.08
$A_{\text{FB}}^{0,\ell}$					1	$R_c^0$					1	-0.18

Table 2: Correlation matrices for observables determined by the  $Z$  lineshape fit (left), and by heavy flavour analyses at the  $Z$  pole (right) [56].

# Top mass dependence on Event Kinematics



- Difficult to define a pole mass for heavy, unstable and colored particle.
- The top mass extracted in hadron collisions is not well defined below a precision of  $O(\Gamma_t) \sim 1 \text{ GeV}$
- Single top decays before hadronizing. To have colorless final states, additional quarks needed.
  - Non-perturb. color-reconnection effects in fragmentation.
  - Ambiguities in top mass definition
- Result:  $m_t^{\text{exp}} \not\equiv m_t^{\text{pole}}$ , and event-dependent.
- With additional theo. uncertainty of 0.5 GeV on  $m_t$ :
  - $M_H = 90^{+34}_{-21} \text{ GeV}$ ,  $M_W = 80.359 \pm 0.013 \text{ GeV}$ ,  $\sin^2\theta_{\text{eff}}^l = 0.23148 \pm 0.00010$ .
  - Only small deterioration in precision.



# Extended STU results



- Several extended STU parametrizations available
  - Here: STU +  $\delta\epsilon_b$ , latter parameter describing  $Z \rightarrow b\bar{b}$  vertex
- SM:  $M_H = 125.7$  GeV,  $m_t = 173.2$  GeV
  - This defines  $(S, T, U) = (0, 0, 0)$
- S, T depend logarithmically on  $M_H$

- Fit result:

$$S = 0.00 \pm 0.10$$

$$T = 0.00 \pm 0.12$$

$$U = 0.06 \pm 0.10$$

$$\Delta\epsilon_b = (2.4 \pm 1.4) \times 10^{-3}$$

	$S$	$T$	$U$	$\delta\epsilon_b$
$S$	1	+0.89	-0.56	-0.13
$T$		1	-0.81	-0.21
$U$			1	+0.20
$\delta\epsilon_b$				1

- (Stronger constraints from fit with  $U=0$ .)

# Prospects for LHC, ILC and ILC with Giga-Z



- Assumed experimental improvements for prospective study:
  - LHC:  $M_W, m_{top}$
  - ILC:  $M_W, m_{top}$
  - Giga-Z:  $M_W, m_{top}, \sin^2\theta_{eff}^l, R_{lep}$
  - ISR-based (BABAR) and BESIII, KLOE-II cross-section measurements, should improve  $\Delta\alpha_{had}(M_Z)$

Quantity	Expected uncertainty			
	Present	LHC	ILC	GigaZ (ILC)
$M_W$ [ MeV]	23	15	15	6
$m_t$ [ GeV]	1.3	1.0	0.2	0.1
$\sin^2\theta_{eff}^l$ [ $10^{-5}$ ]	17	17	17	1.3
$R_\ell^0$ [ $10^{-2}$ ]	2.5	2.5	2.5	0.4
$\Delta\alpha_{had}^{(5)}(M_Z^2)$ [ $10^{-5}$ ]	22 (7)	22 (7)	22 (7)	22 (7)
$M_H (= 120 \text{ GeV})$ [ GeV]	$^{+54}_{-40} \left( ^{+51}_{-38} \right) \left[ ^{+38}_{-30} \right]$	$^{+45}_{-35} \left( ^{+42}_{-33} \right) \left[ ^{+30}_{-25} \right]$	$^{+42}_{-33} \left( ^{+39}_{-31} \right) \left[ ^{+28}_{-23} \right]$	$^{+26}_{-23} \left( ^{+20}_{-18} \right) \left[ ^{+8}_{-8} \right]$
$\alpha_s(M_Z^2)$ [ $10^{-4}$ ]	28	28	28	6

Input from: [ATLAS, Physics TDR (1999)] [CMS, Physics TDR (2006)] [A. Djouadi et al., arXiv:0709.1893][I. Borjanovic, EPJ C39S2, 63 (2005)] [S. Haywood et al., hep-ph/0003275] [R. Hawkings, K. Mönig, EPJ direct C1, 8 (1999)] [A. H. Hoang et al., EPJ direct C2, 1 (2000)] [M. Winter, LC-PHSM-2001-016]